

APPLICATION OF THE FINITE ELEMENT METHOD TO MODEL THE NONLINEAR VOICE COIL MOTION PRODUCED BY A LOUDSPEAKER MAGNET ASSEMBLY.

Mark Dodd
Celestion International & KEF Audio (UK) Ltd.

1. INTRODUCTION

Moving coil transducers rely on the interaction between an electric current and magnetic field to produce a driving force. Past work on the non-linear behaviour of the magnet and voice coil has, with one notable exception [1][2][3], involved retrospective analysis using transducer parameters derived from physical measurements. This work on non-linearity is based on the well-known differential equations used in lumped element analysis with parameters expressed as power series expansions. While these retrospective analyses throw some light on the mechanisms resulting in the distortion produced on existing transducers, they are not helpful in modelling transducers with new motor topologies.

The Finite Element Method (FEM) discussed below is applied here to the modelling of a new loudspeaker motor assembly to predict the non-linear behaviour. The FEM model used takes into account the effect of eddy currents and includes coil motion. A further refinement is the use of a voltage source so the effects of electro-dynamic damping on coil motion are included. This model allows us to predict harmonic and intermodulation distortion and driver response, including electrical input impedance, using FEM. A further benefit of this method is that it allows physical effects to be visualised and quantified regardless of experimental limitations. The technique described requires the input only the material properties and geometry of the loudspeaker motor to provide this information making it an ideal tool for investigating new loudspeaker motor geometries.

2. MOVING COIL TRANSDUCERS

The physics of loudspeaker motors involves the interaction of magnetic field and electric currents to produce a driving force. Let us consider a cylindrical voice coil, formed by a length of wire l in a magnetic field $B(r,z)$. If a direct current flows through the coil with current density J an axial force F is produced integrated over a volume dU .

$$\vec{F} = \int_U \vec{J} \times \vec{B} dU \quad 1$$

For a radial field of average flux density B and current i this equation simplifies to the familiar equation below:

$$F = Bli \quad 2$$

However, the current flowing through the coil will produce its own magnetic field in the gap, which combines with the field of the permanent magnet to give the total gap flux.

$$\Psi_{Total} = \Psi_{static} + \Psi_{vc} \quad 3$$

Consequently the Ampere force produced by the coil is a function of the current as well as axial position. It's worth noting that superposition does not apply in non-linear materials such as iron, the permanent magnet's field will be affected by the field from coil due to the change in the iron's permeability.

When we consider a sinusoidal current through the voice coil, we must consider the effect of the emf generated across any conductive loop produced as a result of the changing magnetic flux. The magnitude of this emf, $e(t)$ is given by:

$$e(t) = -\frac{d\Psi_{Total}(t)}{dt} \quad 4$$

Firstly, a 'back emf' opposes the driving voltage resulting in the inductive electrical impedance of the voice coil. Secondly, emfs are generated across the single turns formed by the magnet iron work. In the iron region the large cross-sectional area and high magnetic permeability of the iron result in a non-uniform current density. The relatively large magnetic fields produced by the current flow in the highly permeable iron cause regions of the current to repel each other, forcing current flow to be restricted to the surface of the metal. For a sinusoidal current the mean skin depth δ is given by the equation below:

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \quad 5$$

Where μ is magnetic permeability, σ is the electrical conductivity and ω is the angular frequency. We should note that the depth of this region of high current density or 'skin' decreases with high frequencies resulting in a resistance increases with frequency. The effect of these induced currents, known as eddy currents, is to produce an alternating magnetic field opposed to the field from the voice coil. The total field is now composed of three parts:

$$\Psi_{Total} = \Psi_{static} + \Psi_{vc} + \Psi_{eddycurrent} \quad 6$$

This additional alternating field results in a decrease of the 'back emf' across the voice coil. One manifestation of this lower emf is that the voice coil self inductance is reduced. Another manifestation is that the flux modulation is reduced. Although the eddy currents flowing in magnet ironwork have often been regarded as detrimental to loudspeaker performance we can see that the effects have some use in allowing adequate midrange sensitivity and reducing flux modulation. It has been shown in [4] that as a result of these eddy currents in the iron, the electrical impedance of the coil is not that of a pure inductance, rather it is a quasi inductance where the inductive impedance rises proportionally to the square root of $j\omega$.

3. TRANSIENT MAGNETIC FEM

Transient Magnetic analyses uses the full version of Maxwell's equations which includes the effect of the magnetic field varying with time t see Equ. 7 [7][8][9]. Non-linear material properties may be used with an arbitrary time varying source and induced currents may be calculated. The hysteretic properties of the iron magnetization curve are neglected in this FEM analysis, Magnetic hysteresis is discussed in [5] as an additional distortion mechanism in the context of the model in [1][2][3]. By solving the model in sufficiently small time steps the interaction between varying magnetic fields and electric currents may be investigated. For convergence to occur in systems containing permanent magnets it is necessary to start with a static field, this avoids the rate of change of flux and thus induced voltages becoming infinite.

$$\mu_0\sigma\frac{d\vec{A}}{dt} + \nabla \times \left(\frac{1}{\mu_r} \nabla \times \vec{A}\right) = \vec{J}\mu_0 + \mu_0\nabla \times \vec{H}_c \quad 7$$

In the FEM model the source strength of a coil is defined in terms of current, to model a voltage source driving the motor we must couple an electrical circuit to the input of the coil. In the software used the electrical circuit is 'close coupled' to the magnetic problem and further iteration is not required. Note that the coil is static and the motor is in the condition that is most often described as 'blocked'.

The FEM model of the motor we are using has two types of conductors, stranded and solid. The stranded conductor is useful to model a coil made of single strand of fine wire such as a voice coil. In this case the current flow is perpendicular to the element, the current density in this type of conductive region is constant since the current in the wire, by Kirchoff's laws, must also be constant along its length. Conversely a solid conductor allows current to flow in any direction, allowing the magnetic field to influence the path travelled by the current allowing eddy currents to flow within the region modelled. Solid conductors are appropriate to model magnet poles, plates and conductive rings.

When we subdivide the geometry into a mesh for analysis by FEM we must ensure that the change of state variable, vector potential, across each element is not too large. If we consider the current density of an eddy current we can use equation 5 to calculate the nominal thickness of the skin depth. The elements used throughout this paper were second order, producing a parabolic approximation to the field, sufficient elements must be used to model the exponential field variation. At higher frequencies the smaller skin depth requires a finer mesh with more elements and will require somewhat greater solution times. The mesh used in this paper uses geometric parameters to define element sizes in this area as a function of frequency [6]. A further refinement in the mesh is to use rectangular elements that may have a high aspect ratio since the change of current density is small parallel to the steel surface. This greatly decreases the number of elements required and allows very much faster solution times than would be possible with triangular elements. The rectangular elements were produced by the mapped mesh technique described in [6] allowing these elements to be 'bent' round curved surfaces. As an example the part of the mesh used to model the skin region is shown in Fig. 1.

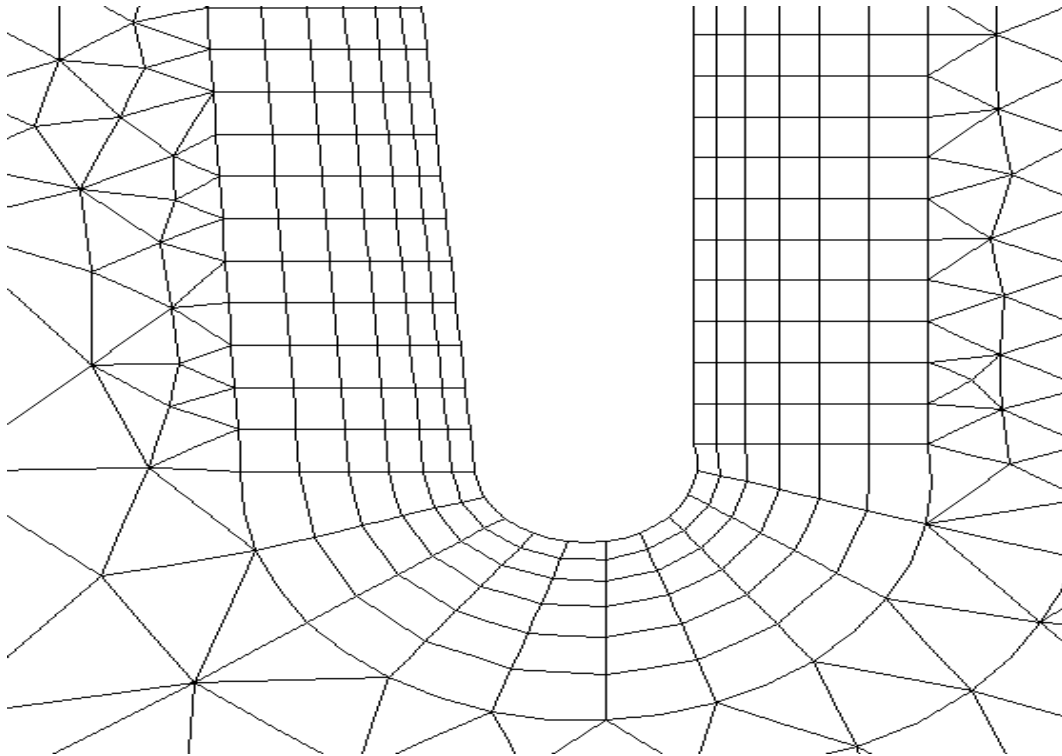


Figure 1. An example of the mesh used to subdivide the geometry into elements for the skin region.

We will now look at some results from the analysis of a new magnet topology. The aim of this topology was to combine the high efficiency of the 'pot' type magnet structure with the high flux density required for a high power low frequency driver. The 100mm diameter voice coil is immersed in a 10mm high magnetic gap. This is energised by two 5mm thick NdFeB rings of 48mm ID & 94mm OD. 1.2 Tesla's of magnetic induction is produced in the gap for a motor weight of 3.7Kg. The magnets are magnetised with opposing polarity so that in effect they are working in parallel. The weight of an equivalent Barium Ferrite structure is 8.8Kg.

The results below were obtained using Flux2D¹, all processing was carried out on a 1.7GHz Pentium 4 PC with 512MB of RDRAM. Solution times with this PC and a well optimised mesh were typically half an hour for each frequency allowing a magnet to be modelled overnight.

A significant benefit of the transient magnetic approach is that the eddy currents may be observed in respect to both their geometric distribution and temporal response. To this end plots of flux and current density isobars in the example magnet assembly is shown for a single time frame in figures 2 & 3 with a 20Hz 40volt sinusoidal signal applied to a coil with 102 turns and a 10.2 ohm DC resistance. The resistance has been increased to allow for the thermal compression occurring in the measured driver.

We can see from these results that the skin depth varies with location, due to the variations of permeability that are shown in figure 4. The permeability of the top plate varies from 1800 to 25 giving skin depth, according to equation 5, between 3.36mm at the pole tips to 0.396mm at the ID of the top plate. The magnet also has some current flow in it and has a calculated skin depth of 30mm, consequently the flux penetrates into the interior of the magnet assembly to the pole. Furthermore the eddy currents are not confined to the region in the poles adjoining the coil, in fact the extent of the skin region in which the current flows depends on the frequency. The current density for a 240Hz input is shown in figure 5, we can see that not only is the skin depth smaller but the region of eddy currents is also smaller.

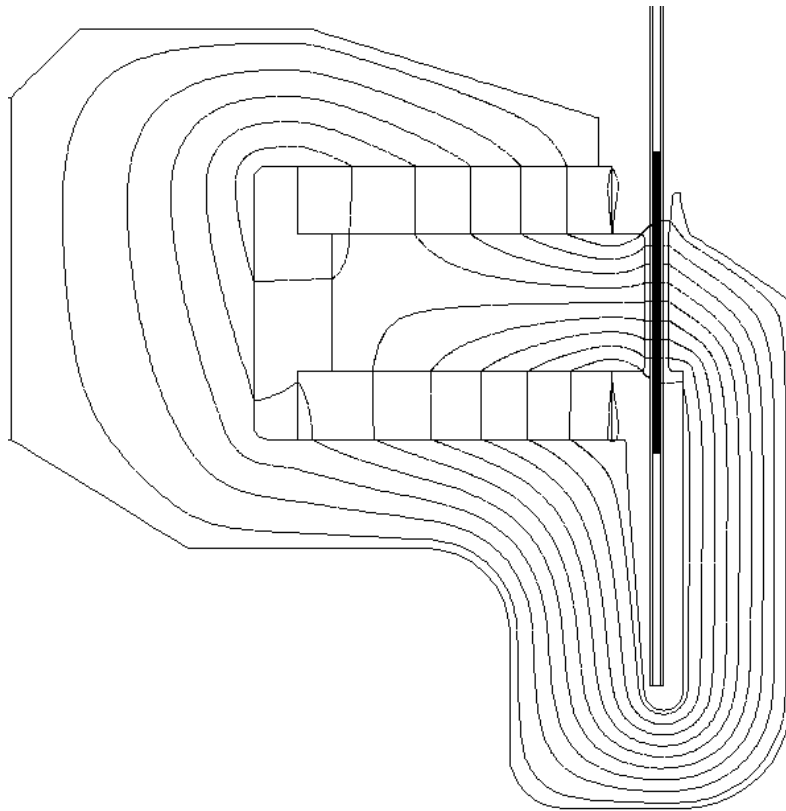


Figure 2 Plot of lines of equal magnetic potential at 20Hz.

1) FLUX2D is the registered trademark of the CEDRAT corporation.

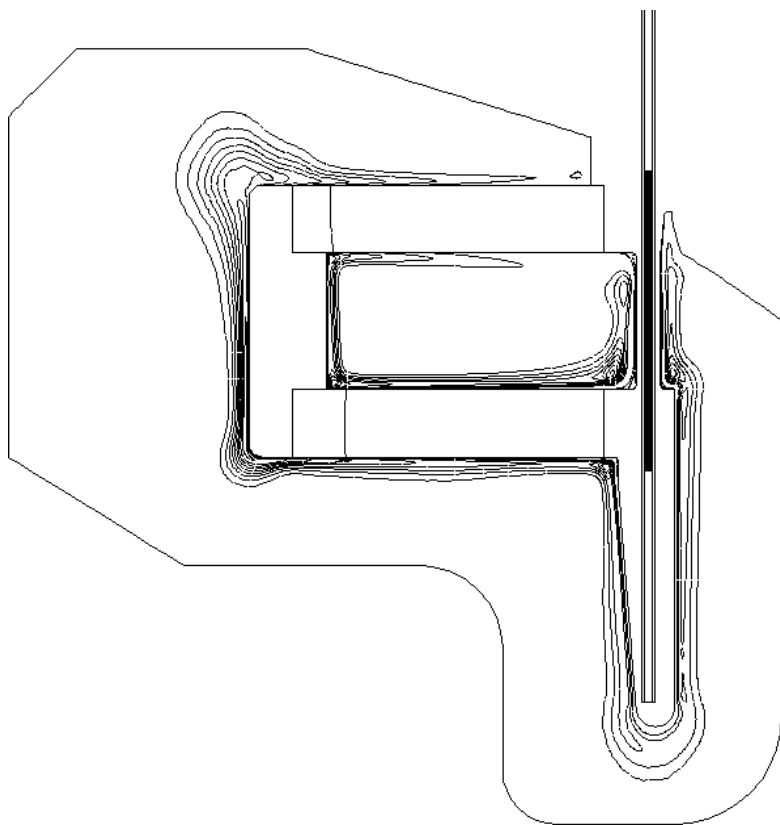


Figure 3 Plot of lines of equal power density at 20Hz.

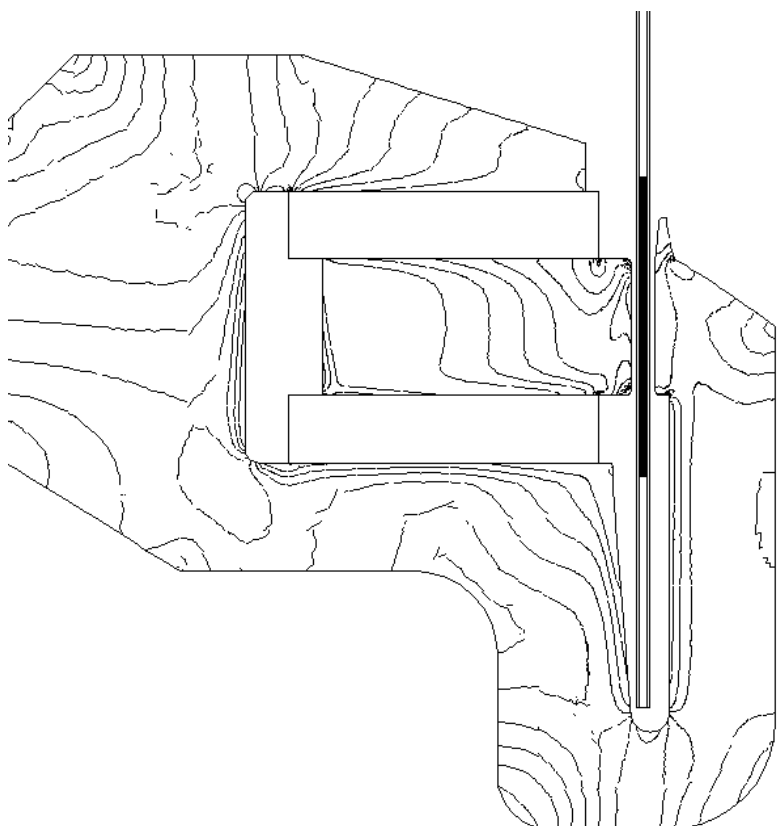


Figure 4 Plot of lines of equal magnetic permeability at 20Hz.

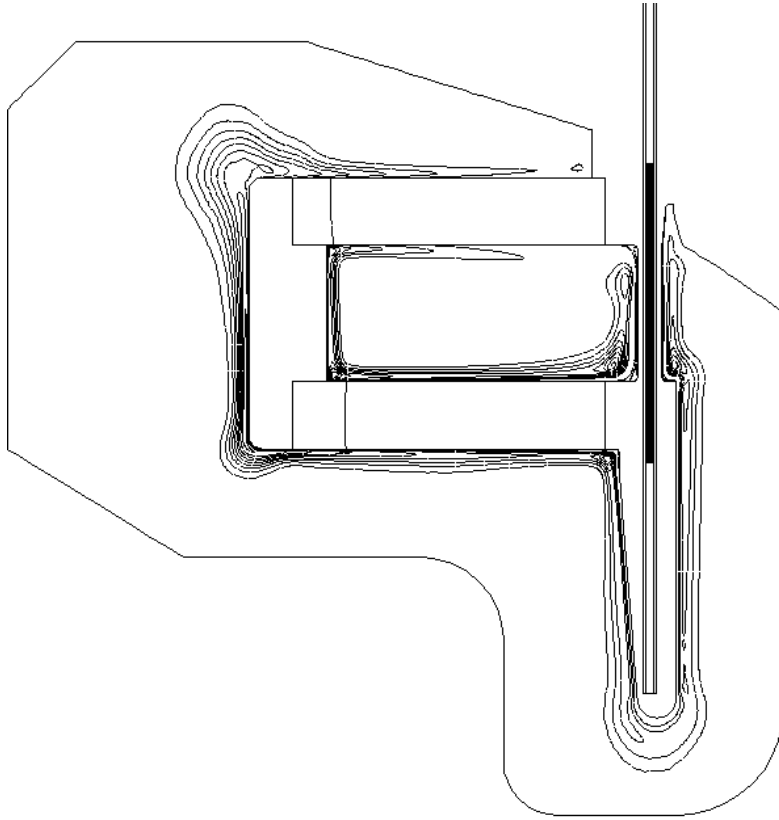


Figure 5 Plot of lines of equal power density at 240Hz.

4. TRANSIENT MAGNETIC FEM WITH KINEMATICS

By coupling Newton's Laws of motion the force produced on the coil may be used to calculate the coil displacement and velocity for consecutive time steps assuming the motor position is fixed and the moving mass is behaving as a rigid body. Errors in the coil motion are kept to a minimum by using the results from the three previous time-steps to derive a third order solution to the coil motion [10]. The mesh used must be able to allow arbitrary coil positions along the axis of motion. The technique used involves creating regions of special mesh shown in figure 6. The displacement regions, in this case are above and below the coil, and the translational regions are on either side. The displacement region is meshed with equal sized quadrilateral elements, movements larger than an axial element length causes the appropriate number of elements to be shifted from one end to the other. The translational regions at the side are auto-meshed with triangular elements. Axial displacement by an integral number of element lengths results in exactly the same mesh for every coil position. For other distances the mesh is stretched by a small amount giving minimal disturbance.

To investigate the accuracy of the motional part of the FEM analysis a model of a moving conductive region in a constant magnetic field was created. By applying a sinusoidal current to the conductor we can investigate the distortion residue produced by the solution process. To simplify the model the no restoring spring and no mechanical losses were included. The differential equations describing the coil motion and model details are shown in appendix 1, these were solved for 15mm displacement to give the required mass and starting velocity. The FEM problem was initiated with the coil at zero displacement and the calculated velocity. The resulting waveform was an almost perfect sine wave with the correct displacement, no drift was observed over the several cycles solved. Harmonics were found to have an amplitude 100dB below the fundamental, much lower than the loudspeaker distortion we will observe. This gives us an indication of the precision from the Kinematic implementation in the software used.

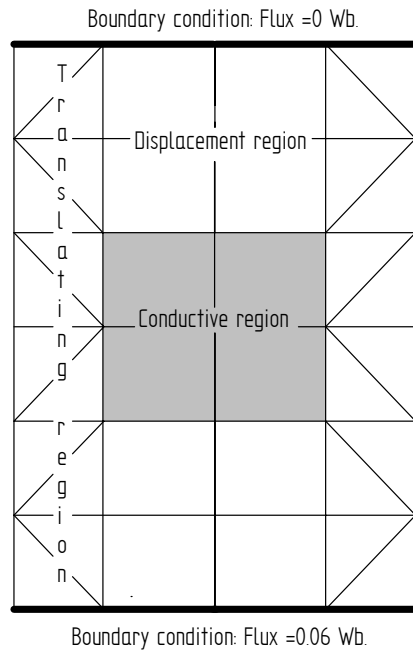


Figure 6 Mesh and boundary conditions for Kinematics test model.

In our FEM model, the current through the voice coil reacts with the total magnetic fields of the permanent magnet and eddy currents in the model to produce the driving force. The method necessitates a large number of steps per period to faithfully reproduce the sinusoidal motion, the work in this paper used between 50 and 120 steps. This model could readily be extended to include a non-linear suspension, however for simplicities sake, this paper is only concerned with the magnetic effects so a linear compliance was used with a spring constant of 5814N/m. Material properties were obtained from manufacturers specifications of similar materials; The moving mass used was 0.102 Kilograms including air load.

Since for a direct radiator the SPL is proportional to the cone acceleration which in turn is proportional to force, for convenience, we have performed the FFT directly on the force. The FFT is executed on the last cycle of the calculated waveform to give us the relative level of the harmonic distortion. The wave form at 20Hz and its corresponding spectrum are shown in Figure 7 for 2.83volt and 40volt inputs. The distortion at this high drive level is very pronounced, it is a result of BL non-linearity, eddy currents, flux modulation and the non-linearity of steel. In common with other FEM solutions the results may be post processed to extract further information, for instance we can also look at the waveform and spectrum other currents flowing in the structure. In figure 8 the waveform and spectrum of the eddy currents flowing in the top plate are plotted for the 40volt 20Hz signal. The high distortion of these currents will result in a distorted magnetic field produced by the eddy currents, in this analysis the effect of the eddy currents on driving force is included rather than being approximated as in previous work. A further result, not illustrated, would be to calculate the dissipated heat in the steel components to evaluate the thermal significance of eddy currents. The results obtained have been used to produce a graph of percentage second and third harmonic distortion over a range of frequencies.

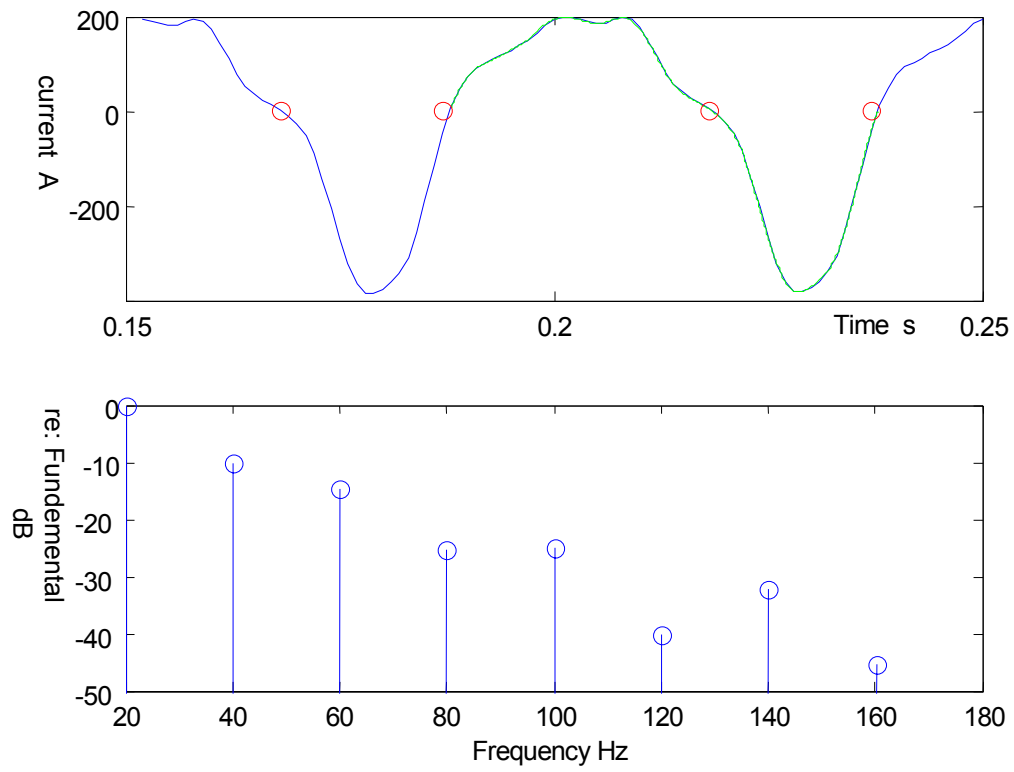


Figure 7 . The calculated waveform and spectrum of the coil force at 20Hz, 40 volts RMS input.

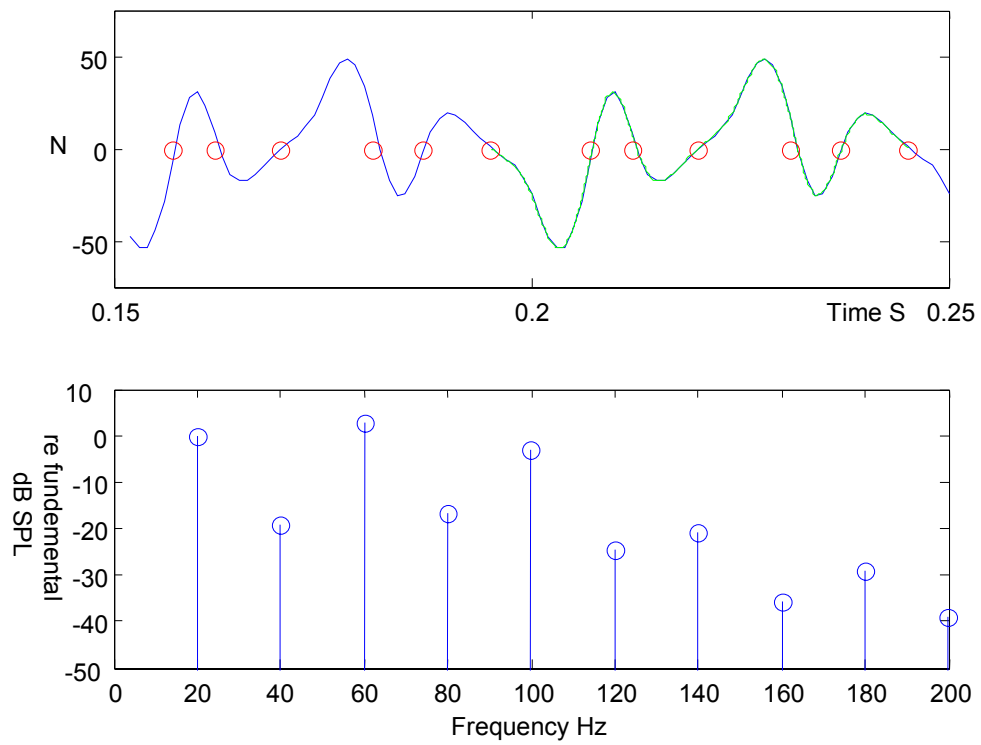


Figure 8. The calculated waveform and spectrum of the eddy currents in the top plate at 20Hz., 40volts RMS.

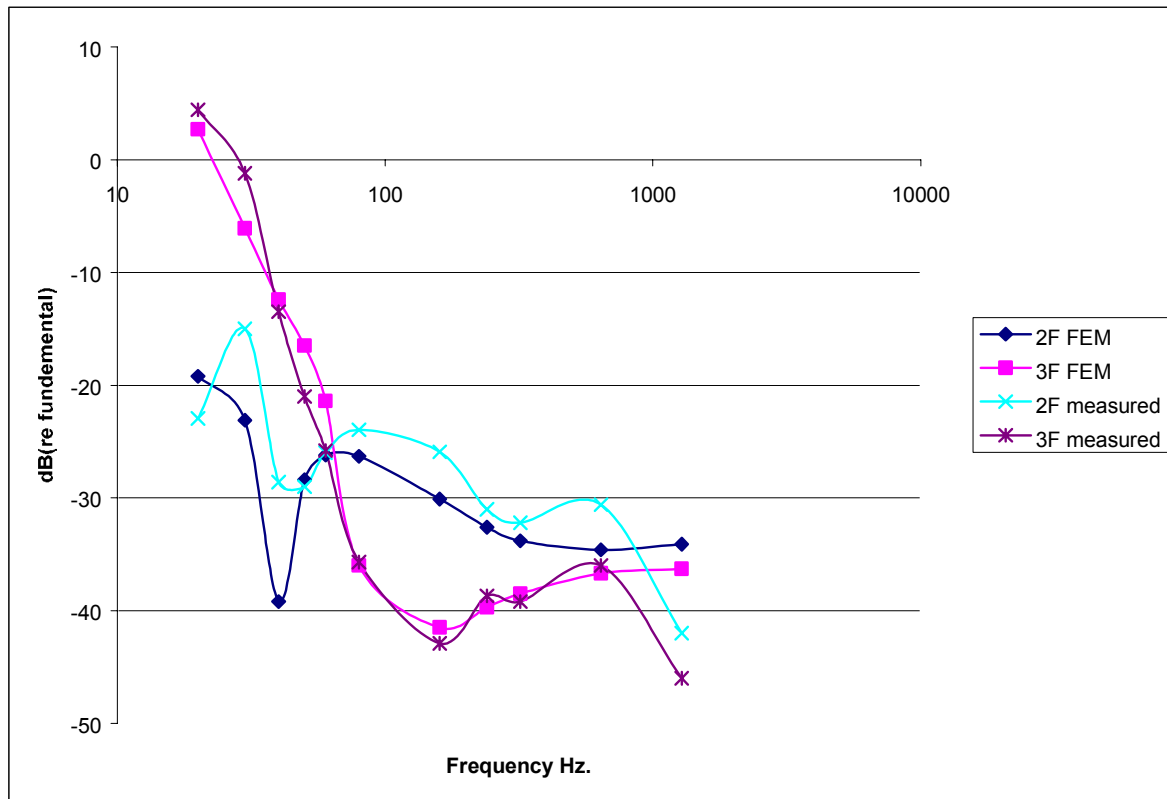


Figure 9. Relative level Second and Third Harmonics from FEM model compared to measured results with 40volts RMS input.

5. CONCLUSION

The direct calculation of harmonic distortion of a loudspeaker motor using Transient Magnetic FEM with Kinematics has the great advantage of requiring only the geometry and material properties of the motor to make the analysis. FEM has the significant advantage of not limiting the geometry of the structure and requiring few assumptions. Very detailed information of magnetic electrical and motional domains is incorporated in the solution, however the computing time and power required limits the number of frequencies and levels analysed.

The results presented here show Transient Magnetic FEM also gives useful data on loudspeaker motor eddy currents. The accuracy of the results has much improved when compared to the original results [11] due to refinements of the mesh and the use of a more linear suspension in the measured driver. The omission of magnetic hysteresis in the model does not appear to be important for the large signal behaviour investigated in this paper. Further work planned will be aimed at refining the results at high frequencies and including the non-linear suspension in the model.

References

- [1] I. Aldoshina, A.Voishvillo,V Mazin. "Loudspeaker Motor Nonlinear Modeling Based on Calculated Magnetic Field Inside the Gap." Presented at the 97th Convention 1994 pre-print 3895.
- [2] I. Aldoshina, A.Voishvillo,V Mazin. "Modeling of Flux Modulation Distortion in Moving Coil Loudspeakers by the Finite Element Method", Presented at the 98th Convention 1995 pre-print 3996.

- [3] A.Voishvillo,V Mazin. "Finite Element Method Modeling of Eddy Currents and Their Influence on Nonlinear Distortion in Electrodynamic Loudspeakers", Presented at the 99th Convention 1995 pre-print 4085.
- [4] J.Vanderkooy. "A model of Loudspeaker Driver impedance incorporating Eddy Currents in the Pole Structure". J.Audio Eng. Soc.,vol. 37, N 3,1989, March pp.119-128.
- [5] Victor Y. Mazin "Modeling of Magnetic Hysteresis and its Influence on Harmonic Distortion in Electrodynamic Loudspeakers."
- [6] CEDRAT, FLUX2D 7.50 user manual.
- [7] P. Lombard, G. Meunier, "A general method for electric and magnetic coupled problem in 2D and magnetodynamic domain" IEEE Trans. On Mag., Vol MAG-28, pp 1291-1294, 1992.
- [8] P. Lombard, G. Meunier, "A general purpose method for electric magnetic combines problems for 2D, axisymmetric and transient systems" IEEE Trans. On Mag., Vol MAG-29, pp 1747-1740, 1993.
- [9] E.Vassent, G. Meunier, A. Foggia,J. C. Sabonnadiere. "Simulation of induction machine operation using a step by step finite element method." Journal of Applied Physics, vol67, pp5809 5812, May 1990.
- [10] E.Vassent, G. Meunier, A. Foggia,J. C. Sabonnadiere. "Simulation of induction machine operation using a step by step finite element method coupled with mechanical equation." Journal of Applied Physics, vol67, pp5809 5812, May 1990.
- [11] M Dodd, " The Transient Magnetic Behaviour of Loudspeaker Motors" To be presented at the 111th Convention 2001.

Appendix 1

Kinematic Test Case.

To allow us to define a region of constant flux we will consider a planar region 1m in depth. The model geometry consists solely of an 1m long linear moving conductor with displacement and the translating air gap regions. No compliance or damping is included. We will find the correct mass to give the desired displacement and the correct starting velocity such that the motion should have no starting transient.

A constant radial flux of 1 T is created by assigning two different Dirichlet boundary conditions to the upper and lower horizontal boundaries.

On the lower horizontal boundary: Value of the flux = 0 Wb.

On the upper horizontal boundary: Value of the flux = 0.06 Wb.

Equation 1 for this very simple case leads to:

$$F = BIl$$

A sinusoidal current I applied to the conductive region with magnitude = 1 A, a phase = 0 degree and frequency = 1 Hz. The resulting force is a sinusoid which varies between 1 N and -1 N. We will impose a maximum displacement of 15mm and find necessary mass and resulting steady state velocity. By imposing this calculated values of mass and a starting velocity equal to the steady state velocity to our FEM model we will then evaluate our FEM results.

$$I = \sin \omega t$$

$$m\ddot{y} = F$$

$$m\ddot{y} = F_0 \sin \omega t$$

$$\ddot{y} = \frac{F_0}{m} \sin \omega t$$

$$\dot{y} = -\frac{F_0}{m\omega} \cos \omega t$$

$$y(t) = -\frac{F_0}{m\omega^2} \sin \omega t$$

$$\Delta y_0 = \frac{F_0}{m\omega^2} \quad \Rightarrow \quad m = \frac{F_0}{\Delta y_0 \omega^2}$$

For 1Hz $\omega = 2\pi$

If $F_0 = 1 \text{ N}$

If $\Delta y_0 = 15 \text{ mm}$

$$m = \frac{1}{15e-3 \times (2\pi)^2} = 1.68869 \text{ kg}$$

Initial condition :

$$\dot{y}(0) = -\frac{F_0}{m\omega} \quad \Rightarrow \quad \dot{y}(0) = -\frac{1}{1.68869 \times 2\pi} = -0.0942475 \text{ m/s}$$

The initial stretching/compression of the spring/position at rest = 0 mm.

The weight of the moving part = 1.68869 kg. For a steady state solution we must impose an initial translation velocity equal to -0.0942475 m/s.