CHAPTER 7

NEGATIVE FEEDBACK

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SECTION 1 : FUNDAMENTAL TYPES OF FEEDBACK

(i) Feedback, positive and negative (ii) Negative voltage feedback at the mid-frequency (iii) Negative current feedback at the mid-frequency (iv) Bridge negative feedback at the mid-frequency (v) Combined positive and negative feedback at the mid-frequency (vi) Comparison between different fundamental types at the mid-frequency.

(i) Feedback, positive and negative

Feedback may be applied to any amplifier, at any frequency, and may be either positive or negative. The application is illustrated in Fig. 7.1 where an amplifier, with voltage amplification A, develops a voltage E_0 across its load. Portion of the voltage across the load (βE_0) is fed back in series with the input terminals, so that the input voltage E_i' differs from the voltage (E_i) across the input terminals of the amplifier itself. It is obvious that

$$E_0 = AE_i \text{ and that } E_i' = E_i \pm \beta E_0,$$

the sign in front of βE_0 taking account of phase relationships. The quantity β is sometimes called the transfer coefficient.
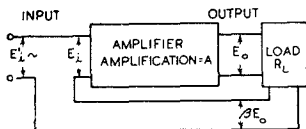

Fig. 7.1. *Block diagram of an amplifier with feedback.*

FIG. 7·1

The effective overall amplification with feedback is therefore

$$A' = \frac{E_0}{E_i'} = \frac{E_0}{E_i \pm \beta E_0}.$$

If the effect of feedback is to increase the gain, the feedback is positive* ; if it decreases the gain, the feedback is negative. Positive feedback is used to convert

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*If βE_0 is much greater than E_i, then the gain will theoretically be reduced irrespective of the sign in front of βE_0.

an amplifying valve into an oscillator (see Chapter 24). Negative feedback is used mainly in amplifiers, both at radio and audio frequencies, although this Handbook covers only low frequency applications.

The voltage βE_0 may be proportional to E_0, in which case it is described as **voltage feedback**; or it may be proportional to the current through the load, when it is called **current feedback**. These two are, of course, identical if the load is a constant resistance, since the voltage and current are then proportional.

Bridge feedback is a combination of voltage and current feedback.

A network may be inserted in the feedback circuit to change the phase of the fedback voltage, or to change its magnitude and/or phase so as to discriminate between different frequencies.

Use of symbols

Black (Ref. A1) and others use the symbols $\mu\beta$ to indicate the same quantity which is here indicated by βA. Some authorities use the symbols $A\beta$, but in certain equations (e.g those involving effective plate resistance) it is necessary to introduce $\beta\mu$ or $\mu\beta$ where μ is the amplification factor of the final valve. The latter form might be confused with Black's $\mu\beta$ which has an entirely different significance. Hence the use of βA in this chapter. A further, but minor, advantage from the use of βA is that the beginner, in considering operation at the mid-frequency, is able to regard the magnitude for β as being simply a fraction.
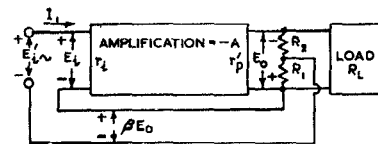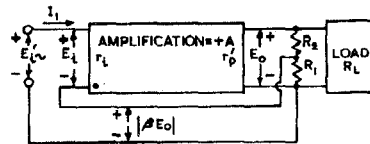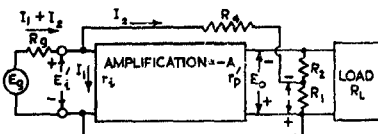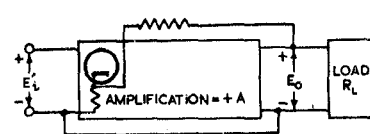

FIG. 7.2A
FIG. 7.2B
FIG. 7.2C
FIG. 7.2D

Fig. 7.2. Block diagrams of amplifiers with negative feedback. It is assumed that $(R_1 + R_2)$ has a resistance very much greater than the load impedance; (A) and (B) have feedback external to the amplifier connected in series with the input voltage (C) has feedback connected in shunt with the input voltage (D) has the feedback connected to a cathode in the amplifier.

(ii) Negative voltage feedback at the mid-frequency

Block diagrams of two conventional forms for applying negative voltage feedback are shown in Figs. 7.2A and B. The polarities marked on the diagrams are instantaneous values at an arbitrary time. In each case the feedback voltage is derived from a voltage divider $R_1 R_2$ across the output and is **applied in series with the input voltage** so as to give negative feedback. In Fig. 7.2A the polarity of the output voltage is opposite that of the input voltage—this is regarded in network analysis as being equivalent to an amplification of $-A$. The amplifier of Fig. 7.2A may consist, for example, of one, or any other odd number of resistance-coupled stages.

In Fig. 7.2B the polarity of the output voltage is the same as that of the input voltage —this being regarded as equivalent to an amplification of $+A$. In Fig. 7.2B the feedback voltage is marked $|\beta E_0|$ thus indicating the magnitude of the voltage and leaving its polarity to be indicated by the positive and negative signs.

In this section the treatment is limited to the mid-frequency at which the amplifier has maximum gain and zero phase shift.

The combined amplifier and feedback circuit, from input back to input, is called the **feedback loop.**

The magnitude of β is equal to $R_1/(R_1 + R_2)$. In all cases, for negative feedback, βA must be negative and the feedback network must be arranged to give this result.

An alternative method of applying the feedback voltage to produce degeneration (Fig. 7.2C) connects the **feedback voltage in shunt with the input voltage.** In this case the amplification must be negative, that is to say there must be an odd number of stages unless a transformer is incorporated (Ref. A17), and β must therefore be positive.

A still further variation is **when the feedback voltage is applied to the cathode of one of the stages in the amplifier** (Fig. 7.2D). In this case the amplification must be positive, for example an even number of r.c.c. stages with the feedback returned to the cathode of the first stage, which must have some impedance between cathode and earth. Alternatively the feedback may be returned to the cathode of the penultimate stage of a multi-stage r.c.c. amplifier. The value of β should be considered as negative on account of the method of connection of the feedback loop.

The following treatment is based on the circuit of Fig. 7.2B but the results may also be applied to any amplifier with the feedback applied externally to the amplifier and in series with the input voltage. It is understood that in all cases the polarity of the feedback voltage is arranged to give negative feedback.

(A) **Gain without feedback**
$$A = E_0/E_i \tag{1}$$
and if feedback is applied and the input voltage increased to E_i' to give the same output voltage E_0 as without feedback, then follows

(B) **Gain with feedback**
$$A' = \frac{E_0}{E_i'} = \frac{E_0}{E_i - \beta E_0} \tag{2a}$$
Combining (1) and (2a),
$$A' = \frac{A}{1 - \beta A} = \frac{1}{(1/A) - \beta} = \frac{A}{1 + [AR_1/(R_1 + R_2)]} \tag{2b}$$
(for chart showing relationship between A' and A see Ref. A21), and if $|\beta A| \gg 1$
then $A' \approx 1/(-\beta)$ \hfill (2c)

The value of β is given by
$$\beta = -R_1/(R_1 + R_2) \tag{3a}$$
The effect of feedback on gain is therefore
$$\frac{A}{A'} = \frac{E_0/E_i}{E_0/(E_i - \beta E_0)} = 1 - \frac{\beta E_0}{E_i} = 1 - \beta A \tag{3b}$$
The quantity βA is negative for negative feedback; $A/A' = 1 - \beta A$ is always greater than unity.

The quantity $(1 - \beta A)$ is called the **feedback factor***. The degree of feedback is indicated by the reduction in gain. For example, an amplifier with " 20 db feedback " is one in which gain has been reduced 20 db by feedback.

In the general case, when the phase angle displacement is not restricted,
$$A' = \frac{A}{\sqrt{1 + |\beta A|^2 - 2|\beta A| \cos \phi}} \tag{3c}$$
[which becomes $A' = A/(1 - \beta A)$ when $\phi = 180°$]
where ϕ = phase angle displacement of amplifier and feedback circuit loop.

(C) **Effect of feedback on harmonic distortion†**
Let the input voltage E_i' be expressed in the form $E_{im}' \cos \omega t$.
Then the output voltage (E_0) will be given by
$$E_{0m} \cos \omega t + E_{2m} \cos 2\omega t + E_{3m} \cos 3\omega t \tag{4}$$

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*Some authorities call βA the feedback factor, and care should therefore be taken in the use of this term.

†Treatment adapted from K. R. Sturley.

where E_{0m} = peak value of fundamental frequency,
  E_{2m} = peak value of second harmonic,
and E_{3m} = peak value of third harmonic.
  The feedback voltage $(-\beta E_0)$ will therefore be
  $-\beta(E_{0m}\cos\omega t + E_{2m}\cos 2\omega t + E_{3m}\cos 3\omega t)$.
  The voltage applied to the input of the amplifier will be
  $E_i' + \beta E_0$
  $= E_{im}'\cos\omega t + \beta(E_{0m}\cos\omega t + E_{2m}\cos 2\omega t + E_{3m}\cos 3\omega t)$
  $= (E_{im}' + \beta E_{0m})\cos\omega t + \beta E_{2m}\cos 2\omega t + \beta E_{3m}\cos 3\omega t$.
  The output voltage will therefore be
  AE_i + harmonic distortion
  $= A[(E_{im}' + \beta E_{0m})\cos\omega t + \beta E_{2m}\cos 2\omega t + \beta E_{3m}\cos 3\omega t]$
  $+ A(E_{im}' + \beta E_{0m})(H_2\cos 2\omega t + H_3\cos 3\omega t)$ (5)
where H_2 = ratio second harmonic to fundamental voltages in amplifier without
      feedback,
and H_3 = ratio third harmonic to fundamental voltages in amplifier without
      feedback.
  But we already have the output voltage in (4) above.
  Equating fundamental components in (4) and (5),
  $E_{0m} = A(E_{im}' + \beta E_{0m})$
  $E_{im}' = (1 - \beta A)E_{0m}/A$ (6)
  Equating the second harmonic components in (4) and (5),
  $E_{2m} = \beta AE_{2m} + A(E_{im}' + \beta E_{0m})H_2$.
  Inserting the value of E_{im}' from (6) above,
  $E_{2m} = \beta AE_{2m} + [(1 - \beta A)E_{0m} + \beta AE_{0m}]H_2$
  Therefore $E_{2m}(1 - \beta A) = E_{0m}H_2$

  Therefore $\dfrac{E_{2m}}{E_{0m}} = \dfrac{H_2}{1 - \beta A}$ (7)

  Similarly for the third or any higher harmonic,

  $\dfrac{E_{3m}}{E_{0m}} = \dfrac{H_3}{1 - \beta A}$ (8)

  That is, the magnitudes of all the harmonics (and of course the intermodulation products) introduced by the amplifier are reduced by negative voltage feedback in the same proportion that the gain is reduced.
  This result is based on the assumptions :
  (1) that E_0 is exactly 180° out of phase, or exactly in phase, with E_i
  (2) that the amplification A is the same for the harmonics as for the fundamental
  (3) that the intermodulation voltages are negligibly small. The presence of intermodulation* has no effect on the second harmonic, but in certain cases the higher order harmonics can increase owing to a small amount of negative feedback ; when the feedback is raised sufficiently these harmonics will decrease again. When $H_2 = 0$, H_3 with feedback will be as indicated by eqn. (8). In any case, the effect is small if the amplifier distortion is initially fairly small.

(D) Effect of feedback on hum or noise introduced by the amplifier

Following a similar method—
  Input voltage $= E_i' = E_{im}'\cos\omega t$
  Output voltage $= E_0 = E_{0m}\cos\omega t + E_{nm}'\cos\omega_n t$ (9)
    where E_{nm}' = peak value of noise or hum output voltage introduced by
          the amplifier with feedback,
    and $\omega_n = 2\pi \times$ frequency of noise or hum voltage.

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*Tellegen, B. D. H., and V. C. Henriquez " Inverse feed-back," W.E. 14.167 (Aug. 1937) 409—quoting R. Feldtkeller ; also correspondence by R. W. Sloane and J. Frommer " Distortion in negative feedback amplifiers " W.E. 14.164 (May 1937) 259 ; 14.166 (July 1937) 369 ; 14.170 (Nov. 1937) 607 ; 15.172 (Jan. 1938) 20.

The feedback voltage is therefore
$$- \beta E_0 = - \beta(E_{0m} \cos \omega t + E_{nm}' \cos \omega_n t).$$
The voltage applied to the input of the amplifier is
$$E_i = E_i' + \beta E_0 = E_{im}' \cos \omega t + \beta(E_{0m} \cos \omega t + E_{nm}' \cos \omega_n t)$$
$$= (E_{im}' + \beta E_{0m}) \cos \omega t + \beta E_{nm}' \cos \omega_n t.$$
The output voltage is therefore
$$E_0 = AE_i + \text{noise voltage without feedback}$$
$$= A(E_{im}' + \beta E_{0m}) \cos \omega t + \beta A E_{nm}' \cos \omega_n t + E_{nm} \cos \omega_n t \qquad (10)$$
Equating the fundamental components in (9) and (10),
$$E_{0m} = A(E_{im}' + E_{0m}).$$
Therefore $E_{im}' = (1 - \beta A) E_{0m}/A$ $\qquad\qquad (11)$
Equating the noise components in (9) and (10),
$$E_{nm}'(1 - \beta A) = E_{nm}$$
$$\frac{E_{nm}'}{E_{nm}} = \frac{1}{1 - \beta A} \qquad\qquad (12)$$
Thus the feedback reduces noise or hum voltages introduced by the amplifier in the same proportion that the gain is reduced, provided the following conditions are fulfilled :

(1) That the signal input voltage is increased with feedback, to maintain the signal output voltage constant.
(2) That β and A have the same values for the hum (or noise) frequency as for the signal frequency.
(3) The voltage fed back does not include any voltages other than the fraction of the amplifier output voltage. In many circuit arrangements this condition is not satisfied ; the most important of these are covered in Sect. 2.

When all noise is considered as originating at the input (e.g. thermal noise), the signal-to-noise ratio is unchanged by the feedback.

(E) Effect of voltage feedback on the output resistance of the amplifier

Every linear network which has a pair of output terminals may, as regards its external effects, be replaced by its equivalent open-circuit voltage in series with its equivalent resistance.
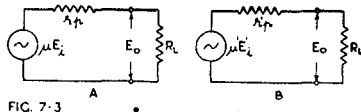

Fig. 7.3. *Equivalent circuit diagrams of amplifier and load resistance* (A) *without feedback* (B) *with feedback.*

FIG. 7.3

The following treatment is based on a single stage, but may be applied to any amplifier provided that the values of μ and r_p are as defined below.

Fig. 7.3A shows the equivalent circuit of the amplifier **without feedback** :
$$E_0 = \mu E_i \left(\frac{R_L}{r_p + R_L} \right) \qquad\qquad (13)$$

$$A = \frac{E_0}{E_i} = \frac{\mu R_L}{r_p + R_L} \qquad\qquad (14a)$$

where μ = amplification factor (i.e. amplification with an infinite load resistance)
and r_p = plate resistance of final stage.

If the amplifier has more than one stage μ must be defined by
$$\mu = A_1 \mu_2 \qquad\qquad (14b)$$
where A_1 = amplification between input and grid of final stage (with due regard to phase reversal)
and μ_2 = amplification factor of final stage.

Fig. 7.3B shows the equivalent circuit of the amplifier **with voltage feedback** :**
$$E_0 = \mu' E_i' \left(\frac{R_L}{r_p' + R_L} \right). \qquad\qquad (15)$$

**Based on B. D. H. Tellegen " Inverse feed-back " Philips Tec. Rev. 2.10 (Oct. 1937) 289.

But $E_0 = A'E_i' = \dfrac{E_i'}{(1/A) - \beta}$ from eqn. (2b).

Applying the value of A from (14a),

$$E_0 = \frac{\mu R_L E_i'}{r_p + R_L(1 - \beta\mu)} \tag{16}$$

This may be arranged in the form

$$E_0 = \left(\frac{\mu.E_i'}{1 - \beta\mu}\right)\left(\frac{R_L}{r_p/(1 - \beta\mu) + R_L}\right) \tag{17}$$

which is the same as eqn. (15) except that

$$\mu' = \frac{\mu}{1 - \beta\mu} \tag{18}$$

and $\quad r_p' = \dfrac{r_p}{1 - \beta\mu}$. $\tag{19}$

Equations (18) and (19) give the effective amplification factor and plate resistance respectively with feedback. It will be seen that these are each equal to the corresponding value without feedback divided by $(1 - \beta\mu)$, whereas the gain, internally produced hum or noise, and distortion are divided by $(1 - \beta A)$. If the amplifier has more than one stage, the value of μ must be as defined by eqn. (14b), while the plate resistance r_p will be that of the final stage.

However, it is important to remember that the actual valve characteristics are not changed by feedback, since feedback is external to the valve ; while the optimum value of load resistance is also unchanged by feedback except under certain very special conditions.

It may be shown that negative voltage feedback as a method for reducing the effective plate resistance, when compared with a transformer giving the same reduction of gain, is more effective than the transformer with low feedback factors, is equally effective with a particular feedback factor, and is less effective than the transformer with feedback factors greater than this value.

Example : 6V6-GT with $\mu = 218$, and $A = 17$.

Gain	$|\beta A|$	$|\beta\mu|$	r_p'/r_p	Transformer impedance ratio
1/2	1	12.8	1/13.8	1/4
1/10	9	115	1/116	1/100
1/100	99	1270	1/1271	1/10000

The " looking-backwards " output terminal impedance of the amplifier is equal to r_p' in parallel with R_L,

$$R_0' = \frac{r_p' R_L}{r_p' + R_L} = \frac{r_p R_L}{r_p + R_L(1 - \beta\mu)} \tag{19a}$$

If r_p' is very much less than R_L, eqn. (17) becomes

$$E_0 \approx \frac{\mu E_i'}{1 - \beta\mu} \tag{20}$$

that is to say, the output voltage is independent of the value of R_L. This is an indication of good regulation.

(F) Effect of voltage feedback on the input resistance

The effect of feedback on the input resistance depends on the method of application of the feedback and not on whether it is voltage or current feedback. For example, consider the amplifier of Fig. 7.2B. Let the input resistance of the amplifier without feedback be r_i.

Then $r_i = E_i/I_1$.

With feedback, the same current will flow, but the input voltage will be

$$E_i' = E_i(1 - \beta A)$$

Therefore $r_i' = E_i'/I_1 = (E_i/I_1)(1 - \beta A)$

Therefore $r_i' = r_i(1 - \beta A)$.   (21)

Thus in this case the input resistance is increased by feedback in the same proportion that the gain is decreased. This applies to all cases in which the feedback voltage is applied in series with the input voltage, including the current feedback case of Fig. 7.4, as will be shown later.

On the other hand, if the feedback voltage is applied in shunt with the input voltage the input resistance will be decreased as proved below for Fig. 7.2C—

The input resistance of the amplifier without feedback is

$$r_i = E_i/I_1.$$

With feedback, the current I_1 will be unchanged, but there will be an additional current I_2 through R_4 and of direction such as to increase the current from the source, which will therefore become $(I_1 + I_2)$. The input voltage applied to the input terminals of the amplifier will be the same as without feedback

Therefore $r_i' = r_i \cdot \dfrac{I_1}{I_1 + I_2}$

which is less than r_i.

Other special cases are covered in Sect. 2.

(iii) Negative current feedback at the mid-frequency

The block diagram of an amplifier with negative current feedback is shown in Fig. 7.4. In this case the feedback voltage is derived from the voltage drop across a resistor R_3 in series with the load impedance. The resistance of R_3 should be very small compared with the load impedance. This diagram also is for the case with the polarity of the output voltage opposite that of the input voltage.
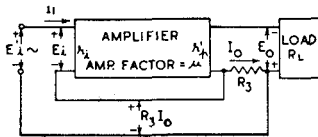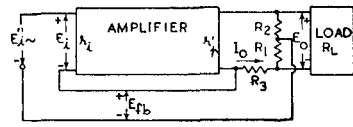

FIG. 7·4   FIG. 7·5

Fig. 7.4.   Block diagram of an amplifier with negative current feedback. The resistance R_3 is very much smaller than the load impedance.
Fig. 7.5.   Block diagram of an amplifier with bridge feedback.

Without feedback but with R_3 in position (Fig. 7.4)

$$A = \frac{E_0}{E_i} = \frac{\mu R_L}{r_p + R_3 + R_L}$$ (22)

With current feedback,

$$A' = \frac{E_0}{E_i'} = \frac{E_0}{E_i + R_3 I_0} = \frac{E_0}{E_i + (R_3/R_L)E_0} = \frac{1}{(E_i/E_0) + R_3/R_L}$$

Therefore $A' = \dfrac{\mu R_L}{(\mu + 1)R_3 + r_p + R_L}$ (23)

Therefore $A' = \dfrac{A}{1 + AR_3/R_L} = \dfrac{A}{1 + \mu R_3/(r_p + R_3 + R_L)}$ (24a)

It is sometimes helpful to substitute

$$- \gamma = R_3/R_L$$

where γ has an application similar to that of β in the case of voltage feedback. Eqn. (24a) may thus be put into the form

$$A' = A/(1 - \gamma A).$$ (24b)

Equation (23) indicates that for a single stage amplifier

(A) **the amplification factor with current feedback** is the same (μ) as without feedback

(B) **the effective plate resistance with current feedback** is

$$r_p' = r_p + (\mu + 1)R_3$$ (25a)

and $r_p'/r_p = 1 + (\mu + 1)R_3/r_p$ (25b)

The increase in plate resistance (eqn. 25b) is proportionally greater than the decrease in gain (eqn. 24a).

When the voltage is fed back over more than one stage, the effective plate resistance with feedback is given by

$$r_p' = r_p + A_1 (\mu_2 + 1)R_3,$$ (25c)

where A_1 = amplification between input and grid of final stage (with due regard to phase reversal)

and μ_2 = amplification factor of final stage.

The "**looking-backwards**" **output terminal impedance** of the amplifier is equal to r_p' in parallel with R_L,

$$R_0' = \frac{r_p' R_L}{r_p' + R_L} = \frac{[r_p + (\mu + 1)R_3]R_L}{r_p + (\mu + 1)R_3 + R_L} \quad \text{for single stage}$$

or $$R_0' = \frac{[r_p + A_1 (\mu_2 + 1) R_3] R_L}{r_p + A_1 (\mu_2 + 1) R_3 + R_L} \quad \text{for more than one stage.}$$

(C) **The ratio of amplification with and without feedback** is given by

$$\frac{A}{A'} = \frac{(\mu + 1)R_3 + r_p + R_L}{R_3 + r_p + R_L}$$ (26)

which may be put into the form

$$A/A' = 1 + A(R_3/R_L) = 1 - \gamma A.$$ (27)

(D) **Approximations when μ is very large**

If μ is very large,

$$A' \approx R_L/R_3 \approx 1/(- \gamma)$$ (28)

which is independent of the amplifier characteristics.

Also, if μ is very large,

$$I_0 \approx E_0/R_L \approx E_i'/R_3$$ (29)

which gives an output current which is constant, irrespective of R_L.

(E) **Input resistance with negative current feedback**

In the circuit of Fig. 7.4—

Without feedback

$$r_i = E_i/I_1$$

With feedback

$$r_i' = \frac{E_i'}{I_1} = \frac{E_i + (R_3/R_L)E_0}{I_1} = \frac{E_i + (R_3/R_L)AE_i}{I_1}$$

Therefore $r_i' = r_i(1 + AR_3/R_L) = r_i(1 - \gamma A)$ (30)

which indicates that with this circuit the input resistance is increased with negative current feedback in the same proportion that the gain is decreased (eqn. 27) and the effect is therefore the same as with Fig. 7.3.

(F) **Harmonic distortion with negative current feedback**

It may readily be shown that, as with negative voltage feedback, the harmonic distortion is reduced in the same proportion that the gain is reduced, subject to the conditions enumerated above (following eqn. 8) :

e.g., $$\frac{E_{2m}}{E_{0m}} = \frac{H_2}{1 + AR_3/R_L} = \frac{H_2}{1 - \gamma A}$$ (31)

(iv) Bridge negative feedback at the mid-frequency

Bridge feedback is a combination of voltage and current feedback, as shown in Fig. 7.5. It is assumed that $(R_1 + R_2) \gg R_L$ and that $R_3 \ll R_L$.

Without feedback : $A = E_0/E_i = \mu R_L/(r_p + R_3 + R_L)$.

With feedback :

Feedback voltage $= R_1 E_0/(R_1 + R_2) + I_0 R_3$

Therefore $E_i' = E_i + R_1 E_0/(R_1 + R_2) + I_0 R_3$

Therefore $A' = \dfrac{AE_i}{E_i'} = \dfrac{AE_i}{E_i + AE_iR_1/(R_1 + R_2) + R_3AE_i/R_L}$

Therefore $A' = \dfrac{A}{1 + AR_1/(R_1 + R_2) + AR_3/R_L}$. (32)

Comparing this with (2b) and (24a), it will be seen that the denominator in (32) includes the second terms of the denominators of both (2b) and (24a). Equation (32) may be put into the alternative forms

$$A' = \frac{\mu R_L}{r_p + (\mu + 1)R_3 + R_L(1 + \mu R_1)/(R_1 + R_2)} \tag{33}$$

and $A' = \dfrac{\mu}{1 + \mu R_1/(R_1 + R_2)} \cdot \dfrac{R_L}{\dfrac{r_p + (\mu + 1)R_3}{1 + \mu R_1/(R_1 + R_2)} + R_L}$ (34)

Equation (34) indicates that—

(1) the amplification factor is reduced by the factor

$\dfrac{1}{1 + \mu R_1/(R_1 + R_2)} = \dfrac{1}{1 - \beta\mu}$ which is the same as for voltage feedback

alone (equation 18).

(2) the effective plate resistance is given by

$$r_p' = \frac{r_p + (\mu + 1)R_3}{1 + \mu R_1/(R_1 + R_2)} = \frac{r_p - (\mu + 1)(\gamma R_L)}{1 - \mu\beta} \tag{35a}$$

Compare this with equation (25a) for current feedback alone and (19) for voltage feedback alone.

If the amplifier has more than one stage, μ must be defined as

$\mu = A_1\mu_2$ (35b)

where $A_1 =$ amplification between input and grid of final stage (with due regard to phase reversal)

and $\mu_2 =$ amplification factor of final stage, thus giving the expression for effective plate resistance

$$r_p' = \frac{r_p + (A_1\mu_2 + 1)R_3}{1 - A_1\mu_2\beta} = \frac{r_p - (A_1\mu_2 + 1)(\gamma R_L)}{1 - A_1\mu_2\beta} \tag{35c}$$

where $\gamma = -R_3/R_L$.

Special applications of bridge feedback (Ref. A9)

(A) The output resistance may be adjusted so as to equal the load resistance. If the voltage and current feedback resistors are adjusted so that

$R_1/(R_1 + R_2) = R_3/R_L$, that is $\beta = \gamma$,

then $r_p' \approx R_L$

provided that $(-\beta\mu) \gg 1$

and that $(-\beta\mu R_L) \gg r_p$.

(B) Negative voltage feedback may be combined with positive current feedback to decrease the plate resistance r_p' to zero or even to make it negative. This is little used in amplifiers because the positive current feedback increases the harmonic distortion. However, it is possible to combine negative feedback in the output stage with positive feedback in an earlier stage to give very useful results—see Sect. 2(xi). This somewhat resembles one form of the balanced feedback in (v) below, except that the design is less restricted.

(C) Negative current feedback may be combined with positive voltage feedback to give very high effective plate resistance.

(v) Combined positive and negative feedback at the mid-frequency

The distortion in a two or three stage amplifier is mainly in the output stage, and the distortion in a well designed first stage will be relatively quite small. It is practicable to apply positive voltage feedback to the first stage only, and then to apply negative voltage feedback over two or three stages in order to secure very low distortion and low output resistance.

This arrangement is shown in the simplified block diagram of Fig. 7.6. The effect of positive feedback on A_1 is to increase its gain from A_1 without feedback to A' with positive feedback. The effect on the whole amplifier, so far as effective plate resistance, distortion in the final stage and gain are concerned, is the same as though A_1 with positive feedback were replaced by another amplifier without feedback but with gain A'. The positive feedback in A_1 will increase the distortion of this stage, but
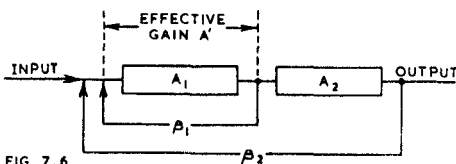
```
        |<--EFFECTIVE-->|
        |    GAIN A'     |
  INPUT |   _____     |      _____     OUTPUT
 ───────┼──|   A₁  |────┼─────|   A₂  |───────────
        |  |_____|    |     |_____|
        |     ┌─β₁─┘     |
        └─────┘          |
  FIG. 7.6          └──β₂─┘
```

Fig. 7.6. Simplified block diagram illustrating one possible arrangement of combined positive and negative feedback.

will decrease the distortion of the whole amplifier provided that the initial distortion in A_1 is small in comparison with that in A_2. A general purpose triode may be used in the first stage, and yet have an effective stage gain greater than that of a pentode with a plate load resistor of 0.25 megohm.

Practical amplifiers using this principle are described in Sect. 2(xi).

Balanced feedback amplifiers

A balanced feedback amplifier is one using both positive and negative feedback in such proportions that the overall gain with feedback is equal to that without any feedback (Ref. A20). There does not seem to be any real advantage in achieving exact balance between the two feedback systems and the designer will generally prefer complete liberty on this point.

(vi) Comparison between different fundamental types at the mid-frequency

Negative feedback :

Voltage Feedback	Current Feedback	Effect
decreases	decreases	gain
decreases	decreases	effect of variations of valves and other components on gain
decreases	increases	effective plate resistance
increases or decreases	increases or decreases	input resistance*
decreases	decreases	harmonic distortion
decreases	increases	effect of load impedance on output voltage
increases	decreases	effect of load impedance on output current
increases	decreases	damping on loudspeaker
increases	increases	frequency response (band width)
decreases	decreases	hum and unwanted voltages introduced in the amplifier (subject to certain conditions).

Positive feedback :

Voltage Feedback	Current Feedback	Effect
increases	increases	gain
increases	decreases	effective plate resistance
increases	increases	harmonic distortion

*The effect on the input resistance depends on the method of applying the input voltage.

Characteristics of amplifiers with negative feedback

Characteristic	No feedback	Voltage feedback	Current feedback†
Voltage gain	$\dfrac{\mu R_L}{r_p + R_L}$	$\dfrac{\mu R_L}{r_p + R_L(1 - \beta\mu)}$	$\dfrac{\mu R_L}{r_p + R_L[1 - \gamma(\mu+1)]}$
or	A	$\dfrac{A}{1 - \beta A}$	$\dfrac{A}{1 - \gamma A}$
Effective amplification factor‡	μ	$\dfrac{\mu}{1 - \beta\mu}$	μ
Effective plate resistance of final stage‡	r_p	$\dfrac{r_p}{1 - \beta\mu}$	$r_p + (\mu + 1)R_3$ $= r_p - \gamma(\mu + 1)R_L$
Input resistance	r_i	$r_i(1 - \beta A)^*$	$r_i(1 - \gamma A)^{**}$
Second harmonic distortion	H_2	$\dfrac{H_2}{1 - \beta A}$	$\dfrac{H_2}{1 - \gamma A}$
Hum or noise introduced by amplifier	E_n	$\dfrac{E_n}{1 - \beta A}$	$\dfrac{E_n}{1 - \gamma A}$

Note : β is negative throughout.
   γ is defined by $-\gamma = R_3/R_L$.

SECTION 2 : PRACTICAL FEEDBACK CIRCUITS

(*i*) *The cathode follower* (*ii*) *The cathode degenerative amplifier and phase splitter* (*iii*) *Voltage feedback from secondary of output transformer* (*iv*) *Voltage feedback from plate—transformer input* (*v*) *Voltage feedback from plate—r.c.c. input* (*vi*) *Voltage feedback over two stages* (*vii*) *Voltage feedback over three stages* (*viii*) *Cathode coupled phase inverters and amplifiers* (*ix*) *Hum* (*x*) *Some special features of feedback amplifiers* (*xi*) *Combined positive and negative feedback* (*xii*) *Choke-coupled phase inverter.*

The idealized conditions assumed in Section 1 do not always hold. At very low and very high audio (or ultrasonic) frequencies, the reactances cause phase angle displacements tending to nullify the "negative" feedback which, in extreme cases, may even become positive. The resulting effects may be evident as poor response to transients (damped oscillations), parasitic oscillations extending over a portion of a cycle of sine waveform or, in extreme cases, oscillation at a very low or very high frequency. This aspect is considered in further detail in Section 3 for those who wish to have anything beyond the most elementary understanding of the problem.

Other complications arise from (a) certain circuit connections affecting the amount of hum, and (b) the application of feedback in parallel with an impedance within the amplifier.

(i) The cathode follower

With a cathode follower the load impedance is connected between cathode and earth instead of between plate and earth in the more conventional arrangement. It is there-

*Applies only to circuits such as Fig. 7.2A or 7.2B in which the feedback voltage is applied in series with the input voltage.

†Over single stage.

‡If the amplifier has more than one stage, the amplification factor must be defined by $\mu = A_1\mu_2$ where A_1 = amplification between point at which feedback is introduced and grid of final stage and μ_2 = amplification factor of final stage.

**Applies only to circuits such as Fig. 7.4 in which the feedback voltage is applied in series with the input voltage.

fore frequently referred to as "cathode loading" in distinction to the conventional "plate loading." As a result of 100 per cent. negative voltage feedback inherent in a cathode follower, both the distortion and the output impedance may be very low.

The basic circuit of a triode cathode follower is shown in Fig. 7.7. It is understood that the input circuit must be conductive to provide a grid return path, and that a suitable grid bias value is achieved by inserting an appropriate battery at, say, point D. The input voltage is applied between grid and earth while the load resistance (R_k) is connected between cathode and earth. The voltage developed across R_k is the output voltage, and the maximum output voltage is slightly greater than for the conventional case with R_k as the plate load resistor. The stage gain, however, is necessarily less than 1 since the input voltage is equal to the grid-to-cathode voltage plus the output voltage (with a medium-mu valve the voltage gain normally approximates 0.9). Since the stage gain is always less than unity, the **input voltage** is always greater than the output voltage. Thus the input voltage may be very much **greater** than the grid bias, but grid current will not flow until the instantaneous difference between the input and the output voltage exceeds the bias.* In some cases the input voltage will be very high as, for example, when a high power output is required from a cathode-loaded low-mu triode operated with a high plate voltage. In extreme cases there may be difficulty in obtaining sufficient voltage in the preceding stage to excite fully the cathode follower.
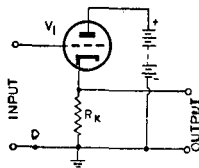

Fig. 7.7. Basic circuit of a cathode follower.

Since a cathode follower does not amplify the input voltage it might, at first sight, be thought that the arrangement fulfilled no useful purpose. Its principal usefulness, however, lies in its impedance characteristics. The input impedance is high while the output impedance (R_0') is low, and the whole device may therefore be regarded as a kind of Impedance Transformer—see (F1) below.

The low output impedance of the cathode follower makes it particularly useful as a driver stage for Class AB₂ or Class B amplifiers, or for pulse techniques where a low impedance source is required.

With the cathode follower, the grid and cathode are in phase with one another, so that there is no reversal of polarity as with plate-loaded amplifiers—this characteristic is sometimes useful in amplifiers, and particularly with pulse techniques.

A cathode follower will work into practically any value of load resistance, although there is a broad optimum value for any particular application.

The effect of interelectrode and wiring capacitances on the gain is generally negligible up to a frequency of 1 Mc/s.

It is important to remember that all electrode voltages must be stated with respect to the cathode ; these will normally have the same values as for plate loading.

The graphical treatment is given on pages 390-394.

(A) Voltage gain at low frequencies

The voltage gain without feedback would be

$$A = \frac{\mu R_k}{R_k + r_p}.$$

The value of the transfer coefficient β will be $\beta = -1$.

---

*More precisely, the difference between the bias and the voltage of the grid current cross-over point.

The **gain with feedback** will therefore be

$$A' = \frac{A}{1 - \beta A} = \frac{A}{1 + A}$$

Therefore $A' = \dfrac{\mu R_k}{(\mu + 1)R_k + r_p}$.   (1a)

which may be written in the form

$$A' = \frac{\mu}{\mu + 1} \cdot \frac{R_k}{R_k + r_p/(\mu + 1)}$$ (1b)

Equation (1a) may also be written in the form

$$A' = \frac{R_k}{\left(\dfrac{\mu + 1}{\mu}\right)R_k + \dfrac{1}{g_m}}$$ (1c)

where g_m = mutual conductance in mhos.

If $\mu \gg 1$, the gain with feedback will be approximately

$$A' \approx \frac{g_m R_k}{1 + g_m R_k}$$ (2)

R_k is the resultant of all resistances between cathode and earth, whether internal or external to the amplifier itself. There is very little increase in gain through making R_k greater than about twice r_p; provided that μ is not less than 10, then the increase of gain through any further increase in R_k is always less than 5 per cent. (It is assumed that the input voltage is small enough to avoid grid current and/or plate current cut-off).

Charts giving gain
Voltage gain (exact) based on μ, R_k and r_p—Ref. C19.
Voltage gain (approx.) based on g_m and R_k for pentodes—Ref. C21.
Gain in decibels (approx.)—Ref. C24.

(B) Effective plate resistance (r_p') at low frequencies
The effective plate resistance in the arrangement of Fig. 7.7, as with any case of voltage feedback, is equal to r_p divided by $(1 - \beta\mu)$, i.e.

$$r_p' = \frac{r_p}{1 - \beta\mu} = \frac{r_p}{1 + \mu} = \frac{1}{g_m} \cdot \frac{\mu}{\mu + 1}$$ (3)

If μ is very much greater than 1,
then $r_p' \approx 1/g_m$ (4a)
with an error not exceeding 5 per cent. if μ is not less than 20.

As with the gain, so too with the effective plate resistance, there is very little change brought about by an increase in R_k beyond $2r_p$, provided that μ is not less than 20. Chart of r_p' (approx.)—Ref. C24.

(C) Equivalent valve characteristics
The results obtained with a cathode follower (e.g. eqn. 1b) are equivalent to those which would be obtained from an equivalent plate-loaded triode* having :
    plate resistance $= r_p' = r_p/(\mu + 1)$
    amplification factor $= \mu/(\mu + 1)$
    mutual conductance $= g_m$ (unchanged)
    and working into a load resistance $R_L = R_k$ (unchanged).

Example 1 : type 6J7 (triode connection)
For 250 volts between plate and cathode, $\mu = 20$, $g_m = 1900$ micromhos and $r_p = 10\,500$ ohms. The value of R_k for greatest power output may be assumed to be approximately equal to $2r_p$, so that a value of 20 000 ohms may be adopted (see Sect. 5 for graphical treatment). The effective plate resistance is therefore
    $r_p' = r_p/(\mu + 1) = 10\,500/21 = 500$ ohms,
and the gain will be

_____

*See Sect. 5(i) for graphical treatment.

$$A' = \frac{20\ 000}{(21/20)(20\ 000) + 525} = 0.93.$$

Example 2 : type 6V6 (triode connection)

With 250 volts between plate and cathode, $\mu = 9.6$, $g_m = 4000$ micromhos and $r_p = 2400$ ohms. If R_k is made equal to 5000 ohms we have

$r_p' = 2400/(9.6 + 1) = 2400/10.6 = 226$ ohms, and the gain will be

$$A' = \frac{5000}{(10.6/9.6)5000 + 250} = 0.866.$$

(D) Distortion

The distortion, within the limits of plate-current cut-off on one hand, and grid current on the other, is reduced by the same factor that the gain is reduced, namely $1/(1 + A)$ where A is the gain without feedback. In a practical case, if $\mu = 20$ and $R_k = 2r_p$, then the distortion is reduced to 3/43 of the distortion with plate loading.

The **maximum input voltage** which may be handled without distortion may be calculated graphically (see Sect. 5).

(E) Calculated operating conditions

The following treatment is based on ideal valve characteristics with straight lines instead of curves, and is therefore only approximate (Ref. C31). It is assumed that the bias is adjusted to allow for maximum voltage swing over the whole loadline.

Operating point :
$$I_b = \tfrac{1}{2} E_{bb}/(r_p + R_k) \tag{4b}$$
$$E_k = \tfrac{1}{2} E_{bb}R_k/(r_p + R_k). \tag{4c}$$
$$E_c = -E_{bb}/(2\mu) \tag{4d}$$

where E_k = voltage across R_k.

If the circuit of Fig. 7.10 is used to provide grid bias :
$$R_2 = (r_p + R_k)/\mu = R_k/A = (1 + R_k/r_p)g_m \tag{4e}$$

where $A = \mu R_k/(r_p + R_k)$

= amplification of valve with plate loading.

Example : Type 6J7 (triode connection)
$$I_b = 125/30\ 500 = 4.1\ \text{mA}$$
$$E_k = 125 \times 20\ 000/30\ 500 = 82\ \text{V}$$
$$E_c = -250/40 = -6.25\ \text{V}$$
$$A = 20 \times 20\ 000/30\ 500 = 13.1$$
$$R_2 = 30\ 500/20 = 1525\ \text{ohms.}$$

In the general case, use may be made of the following table, based on eqn. (4e) :

when	$R_k =$	$0.1/r_p$	$1.0r_p$	$2r_p$	$3r_p$	$4r_p$	$5r_p$
then	$R_2 =$	$1.1/g_m$	$2/g_m$	$3/g_m$	$4/g_m$	$5/g_m$	$6/g_m$

The peak amplitude of the voltage E_k is given by
$$E_{km} = \tfrac{1}{2}E_{bb}R_k/(r_p + R_k) \tag{4f}$$

Special case (1)

If it is desired to use the minimum value of R_k to give a specified peak output voltage E_{km}, the value of R_k is given by
$$R_k = 2E_{km}r_p/(E_{bb} - 2E_{km}) \tag{4g}$$
and the optimum value of bias resistor (Fig. 7.10) is given by
$$R_2 = \frac{E_{bb}}{g_m(E_{bb} - 2E_{km})} \tag{4h}$$

If E_{km} is small compared with E_{bb}, R_2 is thus approximately equal to $1/g_m$.

Special case (2)

If R_k is greater than $1/g_m$ and the whole of R_k is used to provide grid bias, the maximum output voltage is thereby reduced,
$$E_{km} = \frac{E_{bb}R_k}{r_p + (\mu + 1)R_k} \tag{4i}$$

Special case (3)

If R_k is made equal to $1/g_m$ the cathode follower works into a load equal to its own effective plate resistance (r_p'). The value of peak output voltage is given by

$$E_{gm} = \tfrac{1}{2}E_{bb}/(\mu + 1) \tag{4j}$$

and the optimum value of bias resistor R_2 is given by

$$R_2 = \frac{r_p}{\mu}\left(\frac{\mu + 1}{\mu}\right) \approx R_k \tag{4k}$$

so that the valve may be considered perfectly biased by the whole of its cathode resistor R_k.

Special case (4)

When the value of cathode resistance required to provide bias is small compared with $1/g_m$, the arrangement of Fig. 7.11 may be used in which $R_1 = R_k = 1/g_m$, and R_2 is calculated to provide the correct bias. Since R_1 is small compared with $1/g_m = r_p/\mu$, the maximum peak output voltage will be given approximately by

$$E_{km} \approx \tfrac{1}{2}E_{bb}R_1/r_p \tag{4l}$$

Special case (5)

If R_k is small compared with $1/g_m$ and R_k alone used to provide bias, the bias will not be optimum and the maximum peak output voltage will be given by

$$E_{km} = \frac{\mu R_k{}^2 E_{bb}}{r_p[r_p + (\mu + 1)R_k]} = \frac{\mu n^2 E_{bb}}{(\mu + 1)n + 1} \tag{4m}$$

where R_k is neglected in comparison with r_p and $n = R_k/r_p$.

The ratio of the output voltage with correct bias (eqn. 4l) and that with self bias from cathode resistor (eqn. 4m) is given by

$$\text{Ratio} = \frac{2}{1 + 1/\mu n} \tag{4n}$$

(F1) The " looking-backwards " output terminal impedance (R_0')

The impedance, seen when looking backwards from the output terminals of Fig. 7.7 into the amplifier, is equal to r_p' in parallel with R_k. This corresponds to the plate-loaded case where we have to consider r_p and R_L in parallel.

$$R_0' = \frac{r_p'R_k}{r_p' + R_k} = \frac{R_k}{(r_p + R_k)/r_p + g_m R_k} = \frac{1}{g_m + 1/R_k + 1/r_p} \tag{5a}$$

Therefore $R_0' = \dfrac{r_p R_k}{(\mu + 1)R_k + r_p}$ \hfill (5b)

If $r_p \gg R_k$, $R_0' \approx \dfrac{R_k}{1 + g_m R_k}$ \hfill (6)

In special cases, where a very low value of R_0' is required, it may be convenient to obtain it by decreasing the value of R_k, even though this results in a serious reduction in maximum power output. For example, with type 6V6 (triode connection) under the conditions given above, where $r_p' = 226$ ohms,

when R_k = 2400,   1000,   500,   200,   100,   50 ohms.
then R_0' = 206,   184,   156,   105,   69,   41 ohms.

Charts :

R_0' (approx.) based on R_k and g_m—Ref. C23.

R_0' (approx.) but with correction factor to give exact value—Ref. C22.

(F2) Resistance-capacitance coupled cathode follower

An alternative form of load is illustrated in Fig. 7.8 where R_k is the d.c. load resistance, C_c the coupling condenser, and R_g the a.c. shunt load resistance. In cal-
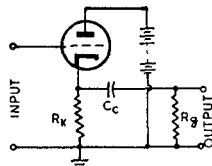

Fig. 7.8. Cathode follower with d.c. load resistance R_k, capacitance coupled to a shunt load resistance R_g.

FIG.7·8

culations regarding grid bias, the resistance of R_k should be used ; in calculations regarding gain the resultant $R = R_k R_g/(R_k + R_g)$ should be used. As for the

similar case with plate-loading, it is usually desirable to have R_g considerably greater than R_k. R_g may take the form of a volume control.

Alternatively, R_k may be selected to suit the grid bias, while R_g is then calculated to provide the desired total a.c. load.

When the circuit of Fig. 7.8 is used as a power amplifier stage, it may be shown (Ref. C32) that maximum power is dissipated in R_g when $R_g = r_p$ and $R_k = \sqrt{2}r_p$, and that when these conditions are fulfilled the power dissipated in R_g is given by

$$P_{max} = E_1{}^2/93r_p$$

where E_1 = plate supply voltage E_{bb} less the voltage where the tangent to the
             $E_c = 0$ characteristic cuts the axis (point B in Fig. 7.8A)

and r_p = plate resistance of valve at junction of d.c. loadline with $E_c = 0$
             characteristic (point D in Fig. 7.8A).

If in Fig. 7.8 the load impedance is removed to a distance and the leads between the cathode follower and the output terminals are lengthened, the "link" between the amplifier and the output may be used as a low impedance loaded a-f line, although it is loaded unequally at the two ends.

The term "link" is used to describe a line which is so short that its characteristics at the frequency of operation may be neglected in comparison with the loading. It should be carefully distinguished from a "transmission line," which has a characteristic impedance determined by the self inductance and capacitance per unit length

FIG. 7.8 A

Fig. 7.8A. Loadlines of circuit Fig. 7.8 used as a power amplifier.

and which, for the suppression of standing waves, requires to be terminated by an impedance equal to the characteristic impedance.

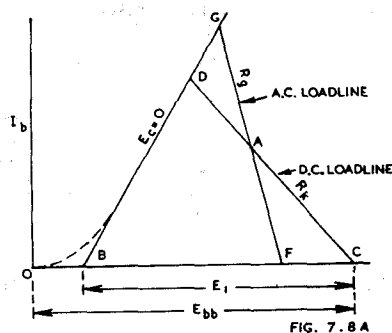(G) **Transformer coupled cathode follower**

With the resistance-coupled case of Fig. 7.7 the plate supply voltage must be higher than the plate-to-cathode voltage and the whole of the power output plus the d.c. losses are dissipated in the resistance R_k. It is possible to replace R_k by the primary of a transformer, and the secondary of the transformer may then be loaded, e.g. by driving the voice coil of a loudspeaker or other electrical network (Fig. 7.9). In this case the full supply voltage, except for the small resistance loss in the transformer primary, is applied between plate and cathode.
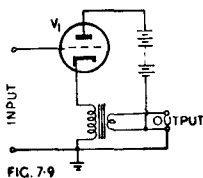

FIG. 7·9

Fig. 7.9. Cathode follower having the primary of a transformer in its cathode circuit, the secondary of the transformer being loaded.

The maximum power output with the same load resistance as for plate loading will be approximately the same. The distortion and the output resistance will, however, be very much lower than for plate loading, and the damping on the loudspeaker will be greater.

A special case of some importance is when V_1 is the driver valve of a Class B power amplifier (see Chapter 13 Sect. 7).

The optimum load resistance may best be calculated graphically (see Sect. 5).

The resistance of the transformer primary will provide at least part of the voltage for grid bias ; the balance may be supplied by a suitable resistance (shunted by the

usual by-pass capacitance) between the cathode and the upper transformer primary terminal.

Alternatively, the primary of the transformer may be connected in place of R_g in Fig. 7.8, and C_c may be adjusted to tune the primary to a suitable low frequency to extend the frequency band to lower frequencies or to give bass boost (Ref. C14).

(H) Grid circuit arrangements and their effect on input resistance and R_0'

If the voltage drop in R_k is greater than the desired grid bias, then a convenient arrangement is to return the grid resistor (R_g) to a tapping point (X) on the cathode load resistor, the position of X being adjusted for optimum operating conditions (Fig. 7.10) ; the bias will be

$$E_{c1} = R_2 I_b \tag{7}$$

where E_{c1} is the bias in volts, R_2 is expressed in ohms and I_b the plate current in amperes. The total load resistance is $(R_1 + R_2)$ which corresponds to R_k in Fig. 7.7. The correct value of C should always be selected on the basis of the input resistance (R_i') and not on the basis of R_g.
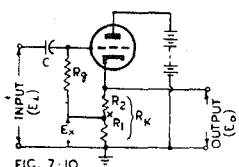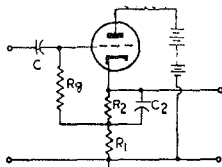

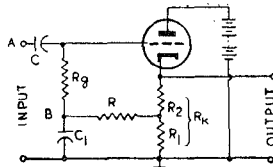FIG. 7·10   FIG. 7·11   FIG. 7·12

Fig. 7.10. Cathode follower with grid bias supplied by a tapping point (X) on the load resistance.

Fig. 7.11. Cathode follower with separate resistances for bias (R_2 by-passed by C_2) and load (R_1).

Fig. 7.12. Cathode follower with alternative form of grid bias circuit.

The circuit of Fig. 7.10 does not apply the full degree of feedback (as in Fig. 7.8) unless the output resistance of the preceding stage is very much smaller than R_g. In other cases the feedback is reduced by a factor $R_0/(R_g + R_0)$ where $R_0 = r_p R_L/(r_p + R_L)$ for the preceding stage.

The **input resistance** (R_i') due to the grid resistor (R_g in Fig. 7.10) is greater than R_g because the voltage across R_g is less than E_i.

Thus $R_i' = E_i/i$

where $E_i =$ the input voltage

and $i =$ the current through R_g (Fig. 7.10).

Now $i = (E_i - E_x)/R_g$

and $E_x = A'E_i R_1/(R_1 + R_2)$.

Therefore $R_i' = \dfrac{E_i}{i} = \dfrac{E_i R_g}{E_i - E_x} = \dfrac{E_i R_g}{E_i[1 - A'R_1/(R_1 + R_2)]}$

Therefore $R_i' = \dfrac{R_g}{1 - A'R_1/(R_1 + R_2)}$ \tag{8}

Special cases :

If $R_2 = 0$, i.e. R_g returned to cathode : $R_i' = R_g/(1 - A')$ \tag{9a}

If $R_1 = 0$, i.e. R_g returned to earth : $R_i' = R_g$ \tag{9b}

As a practical example take type 6J7 (triode connection) with the same conditions as previously (Example 1). The cathode load resistance $(R_1 + R_2)$ is therefore 20 000 ohms. The cathode current (from the published data) is 6.5 mA while the grid bias is −8 volts. The resistance of R_2 is therefore 1230 ohms and R_1 is 18 770 ohms. If R_g is taken as 1 megohm the input resistance will be

$$R_i' = \frac{1}{1 - 0.93 \times 18\,770/20\,000} = \frac{20\,000}{20\,000 - 17\,500} = 8 \text{ megohms.}$$

The **" looking-backwards " output terminal impedance** with the circuit of Fig. 7.10 becomes (Ref. C28)

$$R_0' = \cfrac{1}{g_m \left[1 - \cfrac{R_s}{R_s + R_g} \cdot \cfrac{R_1}{R_1 + R_2}\right] + \cfrac{1}{R_1 + R_2} + \cfrac{1}{r_p}} \tag{10}$$

where $R_s =$ source impedance of input voltage.

[Compare eqn. (10) with eqn. (5a) which applies to the circuit of Fig. 7.7.]

If R_s is not greater than one fifth of R_g, the effect on the output terminal impedance will usually be negligible.

If R_2 in Fig. 7.11 is by-passed, the a.c. load is lower than the static load resistance, which is usually undesirable unless R_2 is very much smaller than R_k. For example, if R_1 is at least 10 times the value of R_2, the effect is usually so small as to be negligible. The input resistance is

$R_i' = R_g/(1 - A')$ which is the same as (9a).

The same remarks apply as for Fig. 7.10 with regard to the output resistance of the preceding stage.

An alternative circuit is shown in Fig. 7.12 in which the reactance of C_1, at the lowest frequency to be amplified, should be small compared with the resistance of R. The input resistance is approximately equal to R_g, provided that $(R + R_1)$ is much larger than the reactance of C_1. With this circuit, the full degree of feedback is obtained irrespective of the output resistance of the preceding stage.

Another method of securing the correct grid bias voltage is to connect a positive grid bias to counteract the too-great negative bias due to R_k, either at the point D in Fig. 7.7 or at the low potential end of the grid resistor R_g. One method of obtaining the negative bias without requiring a separate bias voltage source is shown in Fig. 7.13 where R_3 and R_g form a voltage divider across the plate voltage source. The input resistance here is equal to R_g and R_3 in parallel.

Direct coupling to a cathode-follower is shown in Fig. 7.14. If the first valve has low plate resistance, it would be practicable to drive the cathode follower into the grid current region. The voltage drop in R_{L1} must equal the grid-plate voltage of the cathode follower.
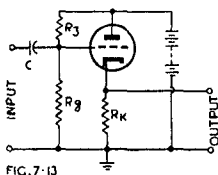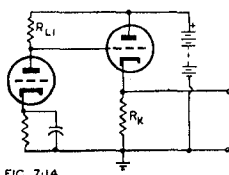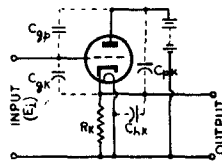

FIG. 7·13   FIG. 7·14   FIG. 7·15

Fig. 7.13. Cathode follower with positive grid bias from voltage divider across B+, in order to counteract the too-great negative bias from R_k.
Fig. 7.14. Direct coupling to the grid of a cathode follower.
Fig. 7.15. Triode cathode follower showing capacitances.

(I) Input capacitance (purely resistive load)

The capacitance from grid to plate (C_{gp}) is effectively across the input (Fig. 7.15). The effect of the capacitance from grid to cathode is reduced by the ratio of the voltage from grid to cathode to the voltage from grid to earth. Thus

$$C_i' = C_{gp} + (1 - A')C_{gk} \tag{11}$$
$$\text{or} \quad C_i' = C_{gp} + C_{gk}/(1 + A) \tag{12}$$

where A has the same meaning as with plate loading, being the stage gain without feedback, and A' is the actual stage gain, being always less than 1 (eqns. 1a and 1b). Note the reversed Miller Effect whereby the C_{gk} is reduced by the factor $1/(1 + A)$.

In a typical case, using type 6J7-GT (triode connection) :—

$A' = 0.93$; $C_{gp} = 1.8 \ \mu\mu F$; and $C_{gk} = 2.6 \ \mu\mu F$.
$C_i' = 1.8 + 0.07 \times 2.6 = 1.8 + 0.18 = 1.98 \ \mu\mu F$

This may be compared with an input capacitance of 22.4 $\mu\mu F$ under equivalent plate loaded conditions.

(J) Output capacitance

The capacitance from plate to cathode (C_{pk}) and also the capacitance from heater

to cathode (C_{hk}) are across the output (Fig. 7.15). The voltage across the capacitance from grid to cathode (C_{gk}) is $(1 - A')e_i$ and the current through C_{gk} is, therefore, $(1 - A')e_i.j\omega C_{gk}$. The current which would flow through C_{gk} if connected directly across R_k is $A'e_i.j\omega C_{gk}$. Therefore, the effect of C_{gk} connected between grid and cathode is the same as that of a capacitance of $[(1 - A')/A']$ $C_{gk} = C_{gk}/A$ connected across R_k. It is assumed that the source of input voltage has a resistance and a reactance each negligibly small in comparison with the reactance of C_{gk}. With this assumption the effect of the capacitance from grid to plate (C_{gp}) is zero, since it merely shunts the input voltage.

Thus, the output capacitance (C_0') under the assumed conditions is given by

$$C_0' = C_{pk} + C_{hk} + C_{gk}/A \qquad (13)$$

This capacitance is effectively shunted across R_0' (see eqn. 5).

In the case of type 6J7-GT (triode connection), $C_{pk} = 17.0$ $\mu\mu$F, $C_{hk} = 10.5$ $\mu\mu$F (approx.), $C_{gk} = 2.6$ $\mu\mu$F, and $A' = 0.93$ (see Example 1).

Therefore $C_0' = 17.0 + 10.5 + 2.6 (0.07/0.93) = 27.7$ $\mu\mu$F.

This is effectively in shunt with the output resistance (R_0') of 447 ohms. In the plate-loaded case there would be a capacitance of 17.0 $\mu\mu$F shunted across 6900 ohms.

In all practical cases, both for input and output capacitances, it is necessary to make allowances for stray capacitances in addition to the valve capacitances.

(K) Special considerations with pentodes

The discussion so far has been principally confined to triodes, or pentodes connected as triodes. It is readily seen that if a pentode is operated with the screen voltage equal to the plate voltage, without special precautions to isolate the screen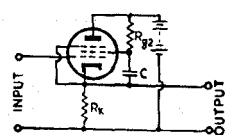 and plate, the result is equivalent to triode operation. The reason is that there is, in the cathode circuit, an impedance which is common to both plate and screen currents, so that the signal voltage drop across it affects the voltages of both plate and screen. Under these conditions the valve ceases to behave like a pentode.

FIG. 7·16

Fig. 7.16. Pentode cathode follower with screen by-passed to cathode.

If it is desired to obtain pentode operation, the voltage between screen and cathode must be maintained steady and free from signal frequency fluctuations. This may be done by connecting an impedance (either a voltage dropping resistor or a choke) between the plate and the screen, and by-passing the screen to the cathode by means of a sufficiently large capacitance (Fig. 7.16).

The screen dropping resistor R_{g2} is effectively in parallel with R_k so far as signal voltages are concerned. Unless R_{g2} is more than 20 times greater than R_k, allowance should be made for its effect.

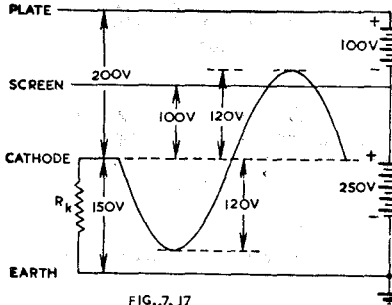If the by-pass condenser (C) is returned to earth instead of to the cathode the result is, again, effectively triode operation except that the screen may be operated at a voltage lower by a fixed amount than that of the plate. For example, if the static voltage from cathode to plate is 200 volts, and from cathode to screen 100 volts, then the voltage from screen to plate is 100 volts (Fig. 7.17). But, when a signal voltage is being amplified, the cathode voltage has a signal-frequency component which may reach a peak value of, say, 120 volts (see diagram). This makes the cathode 20 volts more positive than the screen, or this may be expressed the other way around as the screen being 20 volts negative to the cathode. The result is

FIG. 7. 17

Fig. 7.17. Voltage relationships in pentode cathode follower.

plate-current cut-off and severe distortion, which can only be avoided by a reduced output voltage. In the case of pentode operation, with the screen by-pass condenser returned to the cathode (Fig. 7.16) this type of overloading cannot occur.

When valves having high mutual conductance are connected as triodes, it may be found that parasitic oscillation occurs at a very high frequency. This may be prevented by connecting a 100 to 500 ohm non-inductive resistor in the link between plate and screen, using short leads.

(L) Voltage gain with pentodes

Owing to the very high values of μ with most pentodes, eqn. (2) may be taken as being almost exact :

$$A' = \frac{g_m R_k}{1 + g_m R_k} \tag{2}$$

where g_m = pentode mutual conductance under operating conditions in mhos.

In Example 1, the pentode g_m may be taken as being $(I_b/I_k) \times$ triode g_m ; i.e. $0.8 \times 1900 = 1520$ micromhos.

With $R_k = 20\,000$ ohms,

$$A' = \frac{1520 \times 20\,000 \times 10^{-6}}{1 + 1520 \times 20\,000 \times 10^{-6}} = 0.97.$$

Values of A' up to about 0.99 are practicable with pentodes, using higher values of load resistance.

(M) Input capacitance with pentodes, screen by-passed to cathode (Fig. 7.16)

The input capacitance with pentode operation is different from that with triode operation of the same valve. The capacitance from grid to plate may be neglected, and the total input capacitance is approximately given by

$$C_i' = (1 - A') \times \text{pentode input capacitance} \tag{14}$$

In the case of type 6J7-GT the pentode input capacitance is 4.6 $\mu\mu$F and with a stage gain (A') of 0.97 the input capacitance will be $0.03 \times 4.6 = 0.14$ $\mu\mu$F. This is less than one fourteenth the input capacitance of the same valve with triode connection under similar conditions (previously shown to be 1.98 $\mu\mu$F).

If the screen is by-passed to earth the input capacitance will be the same as for triode operation.

(N) Output capacitance with pentodes (screen by-passed to cathode)

With pentode operation the screen and suppressor are effectively at cathode potential at signal frequency (Fig. 7.16). The " pentode output capacitance " is the capacitance from the plate to all other electrodes except the signal grid and will therefore take the place of the plate-to-cathode capacitance in a triode (eqn. 13). The output capacitance is therefore given by

$$C_0' = \text{pentode output capacitance} + C_{hk}$$
$$+ (1/A) \times \text{pentode input capacitance} \tag{15}$$

In the case of type 6J7-GT with a stage gain of 0.97, the pentode output capacitance is 12 $\mu\mu$F, and the pentode input capacitance 4.6 $\mu\mu$F, so that C_0' is $12 + 10.5 + 0.14 = 22.6$ $\mu\mu$F. This may be compared with 27.7 $\mu\mu$F for triode operation under similar conditions.

(O) Circuit to avoid screen current through the load resistance

With the normal pentode cathode follower, the screen current passes through the load resistance with resulting complications (see also Sect. 5). One way of avoiding these is shown in Fig. 7.18 which produces results identical with those of the cathode follower. The input is applied between grid and plate, the load resistance is in the plate circuit, and the cathode bias resistor has the same value as for normal operation. It has the disadvantage that neither input terminal is earthed, and transformer input is necessary.
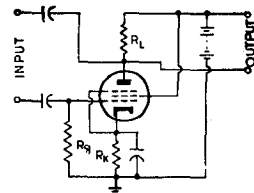

FIG. 7·18

Fig. 7.18. Alternative to the cathode follower, giving similar results, with input applied between grid and plate.

(P) Circuit to make the screen current constant

One other method of avoiding the troubles caused by the screen current involves the use of an additional valve to apply signal voltage of the correct value and phase to the screen, and thereby maintain the screen current constant (Ref. C25).

(Q) Screen-coupled cathode follower

The cathode of a cathode follower may be directly coupled to the screen of the following stage, this being particularly valuable in d.c. amplifiers. See Ref. C26, also Chapter 12 Sect. 9(vi).

(R) " Infinite impedance detector "

The circuit arrangement of the cathode follower is the same as for the so-called " infinite impedance detector " or " reflex detector," but in the latter case it is essential for the plate current to cut off for part of every r-f cycle.

For further information see Chapter 27 Sect. 1(ii)D.

(S) Amplification at all frequencies—complex load

It is impossible in practice to have a purely resistive load owing to the unavoidable effect of the interelectrode capacitances plus stray wiring capacitances.

When the load is a resistance in parallel with a capacitance, the input resistance (and conductance) may become negative, thus leading to oscillation under certain conditions. This may be prevented, where necessary, by the addition of a non-inductive grid-stopper of 100 to 2000 ohms resistance ; in resistance-coupled a-f applications a grid stopper is unnecessary. When an inductance is coupled to the grid circuit (e.g. the secondary of an a-f transformer) it may be necessary to shunt it by means of a fairly low value of resistance to avoid instability.

When the load is inductive the input resistance and conductance are always positive.

The **input reactance** is always capacitive.

References C14, C20.

(T) Gain with capacitive load

When the load impedance is a resistance R_k (as in Fig. 7.16) shunted by a capacitance C_k, the gain is given approximately by (Ref. C6),

$$A' \approx \frac{g_m R_k}{1 + g_m R_k} \sqrt{\frac{1}{1 + \left(\dfrac{f/f_0}{1 + g_m R_k}\right)^2}} \ \underline{/\theta} \tag{16}$$

where $\mu > 20$

f = frequency at which the gain is calculated,

$f_0 = 1/2\pi R_k C_k$

and $\theta = \tan^{-1} -\left(\dfrac{f/f_0}{1 + g_m R_k}\right)$.

The expression within the square root sign in eqn. (16) is known as the **relative gain,** and has been plotted graphically (Ref. C6b, Data Sheet 39). The **phase angle** θ has also been plotted (Data Sheet 41).

References C6, C9 ; Reich (Ref. A30) pp. 166-168 equations 6-34 to 6-34F.

The transient performance under pulse conditions is given in Refs. J1, J5, J6.

(U) Input impedance with capacitive load

Input resistance

$$R_i' \approx \frac{(1 + g_m R_k)^2 + (f/f_0)^2}{\omega C_{gk} R_k g_m (f/f_0)} \qquad \text{when } \mu \gg 1. \tag{17}$$

Input capacitance

$$C_i' \approx C_{up} + C_{gk} \frac{1 + g_m R_k + (f/f_0)^2}{(1 + g_m R_k)^2 + (f/f_0)^2} \qquad \text{when } \mu \gg 1 \tag{18}$$

References C6, C9, C20, J7 ; Sturley (Ref. C34) Vol. 2, pp. 119-120.

(V) Plate resistance with capacitive load

$$r_p' = r_p\left[1 + \frac{g_m R_k}{1 + j(f/f_0)} \cdot \frac{\mu + 1}{\mu}\right] \tag{19}$$

(W) Effect of impedance of input voltage source

This is covered partly by Lockhart (Ref. C6) and more completely by Jeffery (Ref. C9)

(X) Wide band amplifiers

The cathode follower is a valuable method of coupling in wide band amplifiers (Refs. C2, C10, C14).

(Y) Cut-off effect and overloading with capacitive load

When a cathode follower with a capacitive load has a quickly-changing voltage applied to its input, the plate current tends to cut off and positive grid current tends to flow even with input voltages less than those practicable with low frequency sinewave input. At low input voltages it behaves quite normally, and the time constant of its load impedance (R_k and C_k in parallel) is
$R_k C_k r_p'/(r_p' + R_k)$.
When the plate current has been cut off, the time constant becomes $R_k C_k$, which is more than μ times greater (it is assumed that R_k is not less than r_p), and the advantages of the cathode follower are lost. It is therefore essential to ensure that plate-current cut-off never occurs. This effect occurs with a rectangular waveform, a steep sawtooth waveform or with a high frequency sinewave input.
References C13, C15, J3, J4, J5, J6, J9

(Z) Noise in cathode followers

A triode cathode follower has the same reflected noise voltage in the grid circuit, and the same reflected equivalent noise resistance as when used with plate loading. The high input resistance has the effect of improving the signal/noise ratio.

A pentode cathode follower always has a higher equivalent noise resistance than a triode, but the noise resistance is greater when the screen is by-passed to earth than to the cathode.
Refs. B1, B2, C6a (Part 3-A), J12.

(ii) The cathode-degenerative amplifier and phase splitter

With conventional plate loading, the whole load is in the plate circuit ; with the cathode follower the whole load is in the cathode circuit. We now consider the case when part of the load is in the plate circuit and part in the cathode circuit.

Fig. 7.19 shows the general form of the cathode degenerative amplifier with plate load Z_L and cathode load Z_k. The output may be taken from K and earth, or from P and earth, or from both.

(A) Unbypassed cathode resistor

Fig. 7.20 is a particular case commonly known as the unbypassed cathode resistor.*
Although not shown in the diagram, there are valve and stray capacitances which can only be neglected at low audio frequencies ; this condition will be assumed as a first approach. Here R_L is the load resistance, R_k is the unbypassed cathode resistor and R_g the grid resistor. In this application R_k usually has a much lower resistance than R_L. If the signal current through the valve and through R_L is called I_p, then the signal voltage drop across R_k is $R_k I_p$, which is proportional to the signal current through the load ; hence the current feedback. The formulae used below are taken from Sect. 1(iii), putting R_k in place of R_3.
Gain (equation 23, Section 1) :

$$A' = \frac{\mu R_L}{(\mu + 1)R_k + r_p + R_L} \tag{20a}$$

For pentodes

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*For graphical treatment see Sect. 5(v) and (vi).

$$A' \approx \frac{g_m R_L}{g_m R_k + (r_p + R_L)/r_p} \tag{20b}$$

Ratio of gain with and without feedback

From equation (24a) Section 1,

$$\frac{A'}{A} = \frac{1}{1 + \mu R_k/(r_p + R_k + R_L)} \tag{21}$$

or as a rough approximation for triodes

$$A'/A = R_L/(R_L + \mu R_k) \tag{22}$$

An alternative exact form is

$$A'/A = \frac{1}{1 + g_m R_k r_p/(r_p + R_k + R_L)} \tag{23a}$$

or as a rough approximation for pentodes

$$A'/A \approx 1/(1 + g_m R_k) \tag{23b}$$

From equations (24a) and (24b) of Section 1 :

$$\frac{A'}{A} = \frac{1}{1 - \gamma A} = \frac{1}{1 - \dfrac{\mu \gamma R_L}{r_p + R_L(1 - \gamma)}} \tag{24}$$

where $-\gamma = R_k/R_L$.

Example (1) : 6J5 with $\mu = 20$, $r_p = 20\,000$ ohms (under resistance-coupled conditions), $R_L = 100\,000$ ohms, $R_k = 2700$ ohms.
$$A'/A = 1/[1 + 20 \times 2700/(20\,000 + 2700 + 100\,000)] = 0.695.$$
Using the rough approximation, $A'/A \approx 0.65$.

Example (2) : Pentode with $g_m = 2000$ micromhos and
$R_k = 2000$ ohms, $A'/A \approx 1/(1 + 4) \approx 1/5$.

With a pentode, the gain is approximately halved when $R_k = 1/g_m$ (see eqn. 23b). Thus when $g_m = 2000$ micromhos, a value of $R_k = 500$ ohms will halve the gain and halve the distortion.
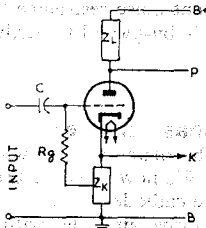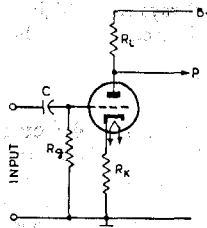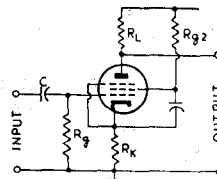

FIG. 7·19   FIG. 7·20   FIG. 7·21

Fig. 7.19.   *General circuit of a cathode-degenerative amplifier.*
Fig. 7.20.   *Resistance coupled amplifier with unbypassed cathode bias resistor, giving cathode degeneration with current feedback.*
Fig. 7.21.   *Resistance coupled pentode with unbypassed cathode bias resistor.*

Input resistance : As the grid resistor R_g (Fig. 7.20) is returned to earth, the input resistance is R_g. If it had been returned to cathode, eqn. (30) Sect. 1 could then be applied, putting R_g in place of r_i :

Input resistance with R_g returned to cathode—

$$r_i' = R_g(1 + AR_k/R_L) = R_g[1 + \mu R_k/(R_L + R_k + r_p)] \tag{25}$$

Effective plate resistance (equation 25a, Sect. 1)

$$r_p' = r_p + \mu R_k \tag{26}$$

and $r_p'/r_p = 1 + \mu R_k/r_p$ \hfill (27)

With the arrangement of Fig. 7.20, the total load on the valve is $R_L + R_k$ and this is the value which should be used in drawing the loadlines on the characteristics. The value of R_k is, however, regarded as fixed, while R_L is regarded as capable of being varied—otherwise the " output resistance " would be meaningless.   In practice,

R_L may be modified by the connection to point P of a coupling capacitance and following grid resistor, thus providing an a.c. shunt load.

If the valve is a pentode, the suppressor is normally connected to the cathode, although it may be connected to earth if the voltage drop across R_k is very small ; the screen should be by-passed to the cathode (Fig. 7.21). In calculating the grid bias, allowance should be made for the d.c. screen current through R_k.

If a less degree of feedback is desired, portion of R_k may be by-passed. In Fig. 7.22, the value of R_k for calculations of gain, r_t' and r_p' should be R_2, but for grid bias should be $(R_1 + R_2)$. Alternatively, R_2 could be by-passed, instead of R_1.
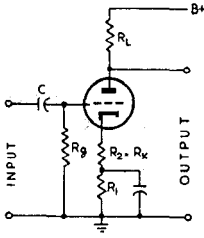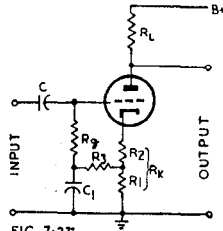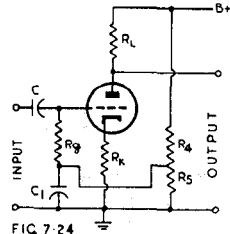

FIG. 7·22   FIG. 7·23   FIG. 7·24

Fig. 7.22.   *Cathode degenerative amplifier with a small degree of current feedback.*
Fig. 7.23.   *Cathode degenerative amplifier with a large degree of current feedback.*
Fig. 7.24.   *Cathode degenerative amplifier with an alternative method of obtaining grid bias.*

If a higher degree of feedback is desired there are several alternatives. In Fig. 7.23 $(R_1 + R_2) = R_k$ for signal frequencies, but only R_2 is effective in producing grid bias. C_1 should have a reactance much lower than $(R_3 + R_1)$. As a slight modification, C_1 may be returned to any tapping point along R_1 and R_2, to give any desired degree of feedback between zero and maximum. An alternative modification is to by-pass R_2, and to return C_1 to any tapping point along R_1. Finally, the arrangement of Fig. 7.24 provides a positive voltage from the voltage divider R_4, R_5 which may be adjusted to give the correct bias voltage from grid to cathode.

When R_L and R_k are not purely resistive, they may be replaced by Z_L and Z_k in the expressions for gain which then become vectors.

Current feedback is undesirable in transformer loaded amplifiers for it tends to stabilize the output transformer's magnetizing current (i.e. make it sinusoidal) and thus produce a distorted output voltage.
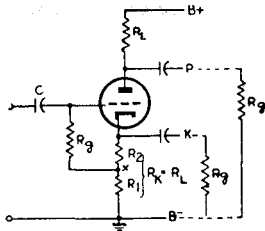

FIG. 7·25

Fig. 7.25.   *Conventional phase splitter.*

(B) **Phase splitter**

If R_k is made equal to R_L, we have the well known " phase splitter " (Fig. 7.25), in which the output from P to earth is equal and opposite in phase to that from K to earth. Obviously $(R_1 + R_2) = R_k = R_L$, and the same signal current (I_p) passes through R_L, r_p, R_2 and R_1. The grid resistor R_g is returned to a point on R_k to give the desired grid bias ($E_c = - R_2 I_b$).

The gain to either output channel is then

$$A' = \frac{\mu R_L}{(\mu + 2)R_L + r_p} \qquad (28)$$

which is always less than 1. In a typical example where $\mu = 20$ and $R_L = 4r_p$, $A' = 0.9$.

The input resistance is

$$' = \frac{R_g}{1 - A'R_1/(R_1 + R_2)} \qquad (29)$$

If $A' = 0.9$ and $R_1 = 9R_2$, then $r_i' = 5.25R_g$.

If R_g is returned to earth, or to a voltage divider, or to a separate bias supply, the input resistance is R_g.

If R_g is returned to cathode, or if R_2 is by-passed, then $r_i' = R_g/(1 - A') \approx 10R_g$.

If R_2 is by-passed, then R_1 must be increased to equal R_L and channel K should be taken from point X. If channel K is taken from point X, and $R_1 = R_L$, then the by-passing of R_2 is unimportant, except that the maximum output voltage will be reduced slightly ; this is of greater importance with low mu valves.

The effective output resistance is different for the two output channels, since P operates with current feedback and K with voltage feedback.

Channel P : $r_p' = (\mu - 1)R_k$ where $R_k = R_L$ (30)
 (from equation 25a, Sect. 1)

Channel K : $r_p' = (r_p + R_L)/(\mu + 1)$ (31)
 (from equation 3)

but this does not affect the balance at either low or high frequencies when the total effective impedance of channel P is equal to that of channel K. The same signal plate current which flows through one impedance Z_P also flows through the other impedance Z_K, and if $Z_P = Z_K$ then the two output voltages are equal. The ratio of the output voltages is $R_L : R_K$ at low frequencies and $Z_P : Z_K$ at any high frequency. The capacitive component of Z_P is the sum of the input capacitance of the following stage, wiring and stray capacitances, and the output capacitance of P channel (eqn. 33), and similarly with Z_K (eqn. 34). The output capacitances of the phase splitter normally differ by only 1 or 2 $\mu\mu F$, having a negligible effect on the balance at 10 000 c/s— see also pages 522-523.

Input capacitance (Fig. 7.25)
 $C_i' = C_{gk}/(1 - A') + C_{gp}(1 + A')$ (32)

Output capacitance
 P channel : $C_0' = 2C_{pk} + C_{gp}(1 + 1/A')$ (33)
 K channel : $C_0' = 2C_{pk} + C_{gk}(1/A' - 1) + C_{hk}$ (34)

The equivalent source impedance that determines the high frequency attenuation for each channel due to shunt capacitance is given by

$$R_0 = \frac{r_p R_L}{r_p + R_L (\mu + 2)}$$ (34a)

and is of the order of 1000 ohms. Hence the excellent high frequency response characteristic (Ref. A33).

(C) Tone control with cathode degenerative amplifier

If R_k (Fig. 7.20) is shunted by a capacitor C_k, it will cause a rise in gain which will reach the maximum of a normal non-degenerative amplifier at frequencies where the reactance of C_k is small compared with R_k. C_k may also be connected across portion of R_k.

If R_k is shunted by an inductor L_k, it will cause a similar rise of gain at low frequencies.

(D) Degenerative cathode impedance $(R_k + L_k)$

A degenerative cathode impedance with R_k in series with L_k may be used with r-f or i-f amplifiers.

(F) Other forms of cathode degenerative amplifiers

Cathode-coupling is described in Section 2(viii).

A cathode-degenerative amplifier may be used as the first or second stage of an amplifier having an overall feedback loop, to provide improved stability and a flatter frequency response characteristic—see Sect. 3(v)E and 3(vii)C.

(iii) Voltage feedback from secondary of output transformer

Voltage feedback* may be taken from the secondary of the output transformer and applied to the grid or any other suitable electrode in the power amplifier valve or any earlier stage. Fig. 7.26 shows one form with feedback to the grid of the power amplifier valve. This is limited to transformer input, and there is a further limitation

*For graphical treatment see Sect. 5(iii) and (iv).

in that the voltage across the voice coil may not be great enough to provide sufficient degeneration. It is important to avoid accidental reversal of the secondary terminals. This, and all its modifications, may be treated as pure voltage feedback following the formulae of Sect. 1, at least in the middle frequency range where β is equal to T_2/T_1, the transformer turns or voltage ratio. It has the properties of maintaining constant the voltage across the secondary, thereby avoiding the effects of transformer leakage inductance at high frequencies, and of reducing the transformer distortion. Unfortunately, at very low and very high audio frequencies, the phase angle introduced by the output transformer tends to cause instability, the tendency being more pronounced as the feedback is applied over 2 or 3 stages. This subject is treated in detail in Section 3.

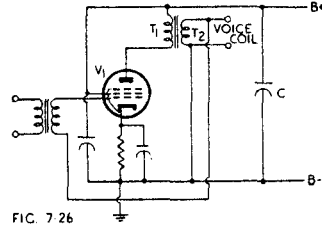*Fig. 7.26. Voltage feedback from secondary of output transformer, over one stage.*

FIG. 7·26

The push-pull form is merely a mirror image of the single-ended variety, and involves two separate secondary windings on the input transformer, and a centre-tap on the secondary of the output transformer, which is earthed.

In either case, if the feedback voltage is greater than desired, the connection may be taken from a voltage divider across the voice coil (or each half of the transformer secondary).
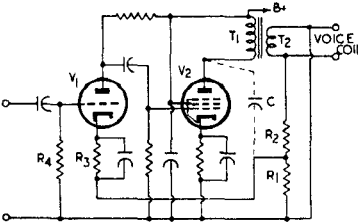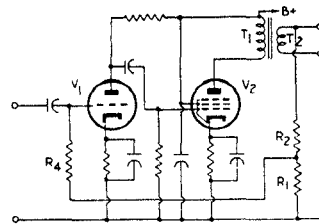

FIG. 7·27

FIG. 7·28

Fig. 7.27.   Voltage feedback from secondary of output transformer to cathode of preceding stage.
Fig. 7.28.   Voltage feedback from secondary of output transformer to grid of preceding stage.

A form suitable for feedback over two stages is shown in Fig. 7.27 where the feedback voltage is reduced by the voltage divider R_1R_2 and applied to the cathode of V_1. The voltage divider total resistance $(R_1 + R_2)$ should be at least 20 times the voice coil impedance, while R_1 should preferably be less than one tenth of R_3, otherwise there will be an appreciable amount of negative current feedback* in V_1. Here

A = voltage gain from grid of V_1 to plate of V_2,

$$\beta = \left(\frac{R_1}{R_1 + R_2}\right)\left(\frac{T_2}{T_1}\right) \tag{35}$$

and $r_t' = R_4$ at mid frequencies (36)

Portion of the plate current of V_1 will flow through the secondary winding. It is sometimes found that a very small condenser C connected from the plate of V_2 to the cathode circuit of V_1 reduces the tendency to instability.

A modified form providing feedback to the grid circuit of V_1 is shown in Fig. 7.28 (note the reversed connections to the secondary of the output transformer). This

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*In practice, this may be desirable.

avoids d.c. through the secondary winding, but the impedance of the preceding stage (R_0) affects the value of β, A being unchanged,

$$\beta = \left(\frac{R_1}{R_1 + R_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{R_0}{R_0 + R_4}\right). \tag{37a}$$

In this case the input resistance is decreased because the feedback voltage is applied in shunt from grid to cathode,

$$r_i' = R_4/(1 - \beta A). \tag{37b}$$

This principle is also incorporated in Fig. 28.3 where feedback is taken from the secondary to a tapping on the volume control.

A further modification, in which the amount of feedback decreases as the volume control is increased, is described in Chapter 35 (Fig. 35.3).

See Sect. 2(vi) below for an analysis of a two stage amplifier with feedback from the secondary of the output transformer.

(iv) Voltage feedback from plate—transformer-input

A typical power output stage with negative voltage feedback from the plate to the bottom end of the secondary of the input transformer is shown in Fig. 7.29. The reactance of C is normally small compared with the total resistance of the voltage divider ($R_1 + R_2$). The condenser C is placed in this position instead of between R_2 and plate since in the latter position the circuit may oscillate owing to the capacitance of C to grid. If there is a tendency to oscillate due to the leakage inductance of the transformer, a small condenser or a resistance, or both, may be connected across the input transformer secondary.
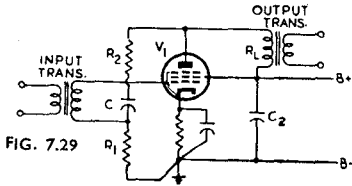

Fig. 7.29. Voltage feedback from the plate to the grid circuit, using transformer-coupled input.

This arrangement reduces the amplitude distortion resulting from saturation of the output transformer core at low frequencies, and improves the low-frequency response. It does not, however, counteract the effect of leakage reactance at high frequencies.

A = voltage gain of valve from grid to plate

$\beta = R_1/(R_1 + R_2)$

and r_i' is extremely high, being unchanged by the feedback (valve input impedance alone).

The same method may be applied to a push-pull stage, but a transformer is required having two separate secondaries, and each valve must have its own feedback network. This circuit is not suitable for use with resistance coupling.

(v) Voltage feedback from plate—r.c.c. input

If it is desired to employ negative feedback from the plate to a resistance-capacitance input coupling, it is impossible to apply the feedback voltage in series opposition to the signal voltage. The only manner of applying it is, so to speak, in shunt with the input signal.

The simplest circuit is Fig. 7.30 in which V_1 is r.c. coupled to V_2, and a feedback path $R_1 C_1$ is provided from plate to grid of V_2. For normal applications C_1 is merely a blocking condenser to isolate grid and plate for d.c. The resistance R_1 acts as a shunt path for signal frequencies in much the same way as the Miller Effect ; the circuit behaves as though an additional shunt resistance equal to $R_1/(A_2 + 1)$ were connected from grid to earth, A_2 being the numerical voltage gain of V_2. This decreased input resistance causes a reduction in overall amplification. The amplification of V_2 is not affected, but V_1 now has a heavy a.c. shunt load, leading to increased distortion and possible overloading. As an example, for the gain to be reduced to

half, the additional a.c. shunt resistance across R_g will be equal to the total resistance of r_{p1}, R_L and R_g in parallel.   A larger than normal value of coupling condenser (C) will obviously be required.

Fig. 7.30.   Voltage feedback from plate to grid, with resistance capacitance coupling. The feedback makes the a.c. shunt load on V_1 considerably less than R_g.
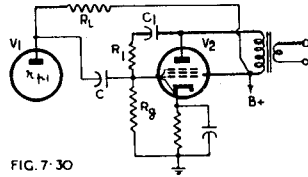
FIG. 7·30

As a modification of Fig. 7.30, R_1 may be made zero and C_1 may have a capacitance of about 10 to 100 $\mu\mu$F to provide degeneration principally at high audio frequencies. This has been used very successfully in small receivers with very little margin of gain, but is only fully effective when no plate by-pass capacitor is used.

An improved circuit is Fig. 7.31 (Refs. E1, E3) in which R_1 and R_2 form a voltage divider across the primary of the output transformer, R_L being connected to the junction (X).   If ($R_1 + R_2$) is at least 10 times the load resistance of V_2, the divider will have little effect on V_2.   It is obvious that the degree of feedback can be adjusted from zero to maximum by moving the tapping point X on the voltage divider from the B+ end to the plate end.   If $R_s + R_L \gg R_1$, where $R_s = r_{p1}R_g/(r_{p1} + R_g)$,

$$\text{then } \beta \approx - \frac{R_s}{R_s + R_L} \cdot \frac{R_1}{R_1 + R_2} \tag{38}$$

An electrically equivalent circuit is Fig. 7.32 (Ref. E2) in which the two resistances R_3 and R_4 take the place of the three resistances R_L, R_1 and R_2 of Fig. 7.31.   For identical operating conditions :

$$R_3 = \frac{(R_1 + R_2)R_L}{R_2} \text{ and } R_4 = \frac{R_3 R_L}{R_3 - R_L} \tag{39}$$

The value of β in Fig. 7.32 is given by

$$\beta = - \frac{R}{R_4 + R} \text{ where } \frac{1}{R} = \frac{1}{r_{p1}} + \frac{1}{R_g} + \frac{1}{R_3}. \tag{40}$$
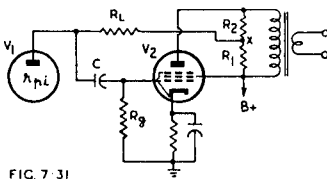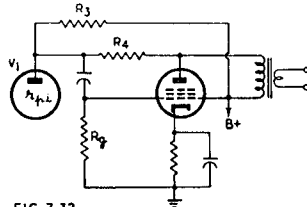
FIG. 7·31

FIG. 7·32

Fig. 7.31.   Voltage feedback applied in series with the load resistor.   The reduction in effective resistance occurs in the load resistor itself.
Fig. 7.32.   Electrically equivalent circuit to Fig. 7.31, using one less resistor.

Both these circuits (Fig. 7.31 and 7.32) have the effect of reducing the effective load into which V_1 works.   This has the merit of extending the response to higher audio frequencies.   In Fig. 7.31 the effective value of R_L becomes
$$R_L' = R_L/(|\beta|A_2 + 1) \text{ where } |\beta| = R_1/(R_1 + R_2)$$
and A_2 is the numerical voltage gain of V_2.   For example if $|\beta| = 0.1$ and $A_2 = 17$, the effective load resistance changes from R_L without feedback to $0.37 R_L$ with feedback.   The load R_L' into which V_1 works is, however, not constant because A_2 varies due to distortion.   This circuit is usually limited to values of $|\beta|$ not greater than say 0.05 to 0.1 for typical applications.

A similar effect occurs with the equivalent circuit Fig. 7.32 in which the feedback causes the effective value of R_4 to change to $R_4/(A_2 + 1)$.   In both circuits V_1 is preferably a pentode, although a high-mu triode may be used with less effectiveness.

Mezger (Ref. E4) has shown that it is equally valid to regard the plate resistance of V_1 (and consequently its amplification factor) as being reduced by feedback, leaving the external resistors unchanged.

(vi) Voltage feedback over two stages

Voltage feedback over two stages permits a wide choice of circuits, without the limitations which frequently arise with a single stage.

(A) One of the deservedly most popular circuits is Fig. 7.33 in which feedback is taken from the plate of V_2 to the cathode of V_1. The most serious limitation to its use is that it cannot be applied without complications* to combined second detector and amplifier valves of the conventional type. It actually involves two types of feedback—the primary voltage feedback, and the subsidiary current feedback caused by the unbypassed cathode resistor in V_1.

This is a form of Duerdoth's multiple feedback—see Sect. 3(v)E—and the two feedback voltages must be added together to determine the performance. The subsidiary feedback increases the stability by reducing the slope of the βA characteristic over a wide range of very low and a wide range of very high frequencies.
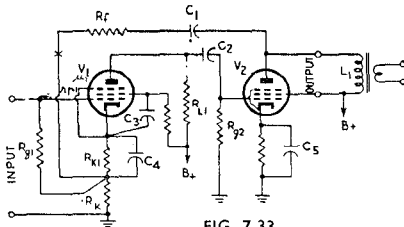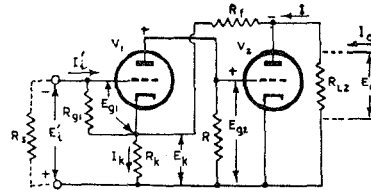

FIG. 7.33   FIG. 7.34

Fig. 7.33. Amplifier with voltage feedback from the plate of V_2 to the cathode of V_1, also incidentally incorporating negative current feedback due to the unbypassed resistor R_k.
Fig. 7.34. Simplified a.c. equivalent of Fig. 7.33 for calculations.

The following theoretical treatment† is based on the simplified a.c. diagram Fig. 7.34. Here V_1 has constants μ_1, g_{m1}, r_{p1} while V_2 has μ_2, g_{m2} and r_{p2}. Resistor R represents R_{L1} and R_{g2} in parallel. It is assumed that, unless otherwise specified, the impedance of the input voltage source (R_s) is zero.

The amplification of stage V_1 without negative current feedback is defined as A_1 where

$$A_1 = E_{g2}/E_{g1} = \mu_1 R/(R + R_k + r_{p1}) \tag{41a}$$

The amplification of stage V_2 is given by

$$A_2 = E_0/E_{g2} = \mu_2 R_{L2}/(R_{L2} + r_{p2}) \tag{41b}$$

Also $\beta \approx R_k/(R_f + R_k)$ $\tag{41c}$

Now $E_0 = A_1 A_2 E_{g1}$ $\tag{41d}$

$E_i' = E_{g1} + E_k = E_{g1} + R_k I_k$ $\tag{41e}$

and $I_k = [\mu_1 E_{g1}/(R + R_k + r_{p1})] + \beta E_0/R_k$ $\tag{41f}$

$= E_{g1}[\mu_1/(R + R_k + r_{p1}) + \beta A_1 A_2/R_k]$ $\tag{41g}$

(neglecting $E_i'/(R_{g1} + R_k)$ as being very small).

Applying these values in equation (41e)

$$E_i' = E_{g1}\left[1 + \frac{\mu_1 R_k}{R + R_k + r_{p1}} + \beta A_1 A_2\right] \tag{41h}$$

Also $E_0 = A_1 A_2 E_{g1} = \dfrac{\mu_1 R}{R + R_k + r_{p1}} \cdot A_2 E_{g1}$

*See Fig. 35.3 for one possible arrangement.

†The Editor is indebted to Mr. E. Watkinson for this method. This method assumes that $(r_{p1} + R)$ is large compared with R_k, and that R_f is large compared with R_{L2}.

$$= \left(\frac{\mu_1 R}{R + R_k + r_{p1}}\right)\left(\frac{E_i'}{1 + \beta A_1 A_2 + \mu_1 R_k/(R + R_k + r_{p1})}\right)A_2$$

$$= \left(\frac{\mu_1 R}{R + R_k + r_{p1}}\right)\left(\frac{A_2 E_i'(R + R_k + r_{p1})}{R + R_k + r_{p1} + \mu_1 R_k + \beta A_1 A_2 (R + R_k + r_{p1})}\right)$$

$$= E_i'\left(\frac{A_2 \mu_1 R}{R + (\mu_1 + 1)R_k + r_{p1} + \beta A_1 A_2 (R + R_k + r_{p1})}\right)$$

Therefore
$$E_0 = E_i' \frac{A_2 \mu_1 R}{\left[R_k\left(\dfrac{1 + \mu_1 + \beta A_1 A_2}{1 + \beta A_1 A_2}\right) + R + r_{p1}\right]\left[1 + \beta A_1 A_2\right]} \qquad (41i)$$

Also when $R_k = 0$ there is no current feedback :

$$E_0 = E_i \frac{A_2 \mu_1 R}{R + r_{p1}} . \qquad (41j)$$

The overall amplification with feedback is given by
$$A' = E_0/E_i' \qquad (41k)$$
while that without feedback is given by
$$A = E_0/E_i = A_2 \mu_1 R/(R + r_{p1}) \qquad (41l)$$

Therefore
$$\frac{A'}{A} = \left[\frac{A_2 \mu_1 R}{\left[R_k\left(\dfrac{1 + \mu_1 + \beta A_1 A_2}{1 + \beta A_1 A_2}\right) + R + r_{p1}\right]\left[1 + \beta A_1 A_2\right]}\right]\left(\frac{R + r_{p1}}{A_2 \mu_1 R}\right)$$

Therefore
$$\frac{A'}{A} = \frac{R + r_{p1}}{\left[R_k\left(\dfrac{1 + \mu_1 + \beta A_1 A_2}{1 + \beta A_1 A_2}\right) + R + r_{p1}\right]\left[1 + \beta A_1 A_2\right]} \qquad (41m)$$

Output impedance

The effect of the unbypassed cathode resistor depends on the resistance of the source (R_s).

Let $\delta = R_g/(R_s + R_g)$ \hfill (42a)

Let voltage E_0 be applied across R_{L2} from an infinite impedance source with no input voltage applied to the amplifier.

Then $I_0 = \dfrac{E_0}{R_{L2}} + \dfrac{E_0}{r_{p2}} + \dfrac{E_0}{R_f + R_k} + \dfrac{E_0 \beta \delta A_1 \mu_2}{r_{p2} + R_{L2}}$ \hfill (42b)

The final term in eqn. (42b) is the effect of the voltage E_0, a fraction of which is fed back across R_k, and amplified by V_1 and applied to the grid of V_2. The effect in the plate circuit of V_2 is the same as though a voltage $E_0 \beta \delta A_1 \mu_2$ were applied to a series connection of r_{p2} and R_{L2}.

Thus the " looking backwards " output terminal impedance is given by

$$R_0' = \frac{E_0}{I_0} = \frac{1}{\dfrac{1}{R_{L2}} + \dfrac{1}{r_{p2}} + \dfrac{1}{R_f + R_k} + \dfrac{\beta \delta A_1 \mu_2}{r_{p2} + R_{L2}}} \qquad (42c)$$

i.e. the effect of the feedback is to add another impedance,
$$\frac{r_{p2} + R_{L2}}{\beta \delta A_1 \mu_2},$$
in parallel with the output, as shown in Fig. 7.41.

Input impedance

$$R_i' = \frac{E_i'}{I_i'} = \frac{E_{g1}\left[1 + \dfrac{\mu_1 R_k}{R + R_k + r_{p1}} + \beta A_1 A_2\right]}{E_{g1}/R_{g1}} \qquad (42d)$$

Therefore $R_i' = R_{g1}\left[1 + \dfrac{\mu_1 R_k}{R + R_k + r_{p1}} + \beta A_1 A_2\right]$ \hfill (42e)

Therefore $\dfrac{R_i'}{R_i} = 1 + \dfrac{\mu_1 R_k}{R + R_k + r_{p1}} + \beta A_1 A_2$ \hfill (42f)

where R_i is defined as the input resistance when $R_k = 0$.

Conclusions

Eqn. (41m) shows that the effect of feedback in reducing gain in this circuit (Figs. 7.33 and 7.34) is not given simply by the product of the current feedback gain reduction and the voltage feedback gain reduction, each considered separately.

Eqn. (41h) indicates that the two feedback voltages are effectively added so far as their effect on gain is concerned. This is in line with Duerdoth's multiple feedback theory in Sect. 3(v)E.

Eqn. (42c) shows that the effective plate resistance of V_2 differs from that of an equivalent amplifier without current feedback only on account of δ. When the source impedance is zero, δ has the value unity, and the effective plate resistance of V_2 is identical with that of an equivalent amplifier with voltage feedback only.

It is obvious that current feedback in V_1 cannot directly affect the plate resistance of V_2. The only indirect effect (through δ) is the result of partial application of the feedback voltage to the grid of V_1, which must also affect the gain. In other words, the impedance of the input voltage source affects the gain, the effective plate resistance and the output resistance R_0'.

If the source is reactive, some non-linear frequency characteristic will occur— for example, if the source is a crystal pickup there will be some measure of bass boosting.

The circuit of Fig. 7.33 tends to produce peaks at low and high frequencies in the response characteristic unless the feedback factor is low. An analysis of the peaks has been made by Everest and Johnston (Ref. H5) based on the ideal condition of zero-impedance screen circuits and unbypassed cathode resistors ; it also omits any complications arising from an output transformer. Methods for reducing or eliminating these peaks are described in Sect. 3(vi) and (vii).

The condenser C_1 in the feedback network of Fig. 7.33 is normally only a blocking condenser ; it may, however, be designed to produce a fixed degree of bass boost. Alternatively, C_1 may remain as a blocking condenser and a second condenser inserted at point X to provide bass boosting ; the latter may be shunted by a variable resistance to form a continuously variable tone control (see Chapter 15 Sect. 2 and Fig. 15.11).

It will be demonstrated in Sect. 3 that feedback over two stages of r.c. coupling is normally stable ; this does not necessarily hold if cathode or screen by-pass condensers are used or if an iron-cored transformer forms any part of the feedback loop.*

(B) Feedback from secondary of output transformer

See Sect. 2(iii) and Figs. 7.27 and 7.28 for the general description of such an amplifier.

An illustration of the effects of negative feedback on a simple 2 stage a-f amplifier can be obtained by calculating† the low frequency response of the circuit of Fig. 7.35. In this simplified circuit batteries are used as voltage sources to minimize calculation, but the effect of cathode by-passes (or of screen by-passes) could be covered without difficulty by using Figs. 12.11 A and B and 12.3 A and C.

The values of components used in interstage couplings have also been chosen for simplicity. Referring to Table 42 in Chapter 38 it will be seen that the reactance of a 0.01 μF condenser (the coupling between the 6AV6 and 6AQ5) at 50 c/s is 318 000 ohms. It is assumed that the series impedance of R_7, and the parallel combination of R_6 and the 6AV6 plate resistance [see Chapter 12 Sect. 2(xiii) Eqn. 15] is also 318 000 ohms, so that at 50 c/s the reactance and resistance of the interstage coupling network are equal.

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*The feedback loop is the complete path, commencing from the point to which the feedback is returned, through the amplifier to the point from where the feedback is taken, back through the feedback network to the starting point.

†The Editor is indebted to Mr. E. Watkinson for the calculation of this example.

Similarly the reactance of the output transformer primary at 50 c/s is equal to the plate resistance of the 6AQ5 in parallel with the reflected load and winding resistances (coupling 3).
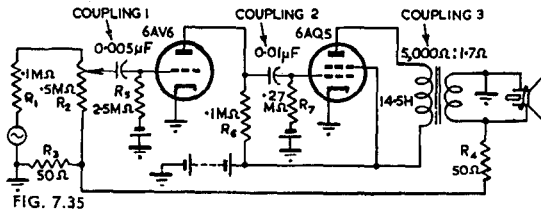

Fig. 7.35. *Simple feedback amplifier in which setting of volume control alters feedback.*

FIG. 7.35

Movement of the input volume control is assumed to have no effect on the frequency ($12\frac{1}{2}$ c/s) at which the reactance of the input condenser is equal to the resistance in series with it, say 2.55 megohms (coupling 1).

The amplification factor, μ, of the 6AV6 is 100 and its plate resistance, r_p, 80 000 ohms, so the mid-frequency 6AV6 gain, A_1 [see Chapter 12 Sect. 2(vii)] is

$$A_1 = \frac{100 \times 73\,000}{80\,000 + 73\,000} = 48$$

where 73 000 ohms is the value of R_6 and R_7 in parallel.

The 6AQ5 μ is 210 and its r_p 52 000 ohms so that the 6AQ5 gain

$$A_2 = \frac{210 \times 5000}{52\,000 + 5000} = 18.4.$$

The output transformer impedance ratio is 5000/1.7 so that its gain

$$A_3 = \sqrt{1.7/5000} = 1/54.$$

Total gain from 6AV6 grid to voice coil :

$A = A_1 \times A_2 \times A_3 = 48 \times 18.4/54 = 16$ times $= 24.1$ dbvg.*

When the volume control is turned to its minimum setting, one half of the output voltage is applied to the amplifier input, i.e. $\beta = -\frac{1}{2}$ and the factor

$$\frac{1}{1 - \beta A} = \frac{1}{1 + 16 \times \frac{1}{2}} = \frac{1}{9}.$$

As the volume control is turned up, a smaller amount of the negative feedback voltage is applied to the 6AV6 control grid, and when the control is 50 000 ohms from its maximum setting only one quarter of the previously used feedback voltage is effective (ignoring the shunting effect of R_5). In this case

$$\frac{1}{1 - \beta A} = \frac{1}{3}.$$

The low frequency response curves will be calculated for each of these volume control settings, but it is first necessary to obtain the response of the amplifier without feedback. This is done in Table 1. Lines 1 and 2 of Table 1 are obtained from Figs. 12.9A and B with $12\frac{1}{2}$ c/s taken as the frequency for 70.7 per cent. voltage gain (when the reactance is numerically equal to the resistance) in line 1, and 50 c/s in line 2. Thus for the 25 c/s response in line 1

$$\frac{\text{Actual frequency}}{\text{Frequency for 70.7 per cent. frequency response}} = 2$$

so that attenuation = 1 db and phase shift = 27°, and so on for each of the other frequencies in the two lines.

Line 3 is obtained by adding twice the attenuation and phase shift of line 2 (because · there are two 50 c/s couplings) to line 1.

Line 4 is the result of adding the mid-frequency gain of the amplifier (24.1 db) to the attenuation and phase shift at each of the tabulated frequencies. This gives the frequency response as plotted at A in Fig. 7.36.

In line 5 the gain A is expressed numerically, giving the actual gain at each frequency.

*Gain expressed in decibels of voltage gain—see Chapter 19 Sect. 1(vi)A.

Table 2 sets out the calculations necessary to obtain the modification of the frequency response by the application of feedback when $\beta = -\frac{1}{2}$. Line 1 results from converting the attenuation expressed in db in line 3 of Table 1 to fractions, and line 2 is obtained by multiplying the mid-frequency gain, the attenuation and phase (for the frequency concerned) by β. The result is βA which is plotted in Fig. 7.37.
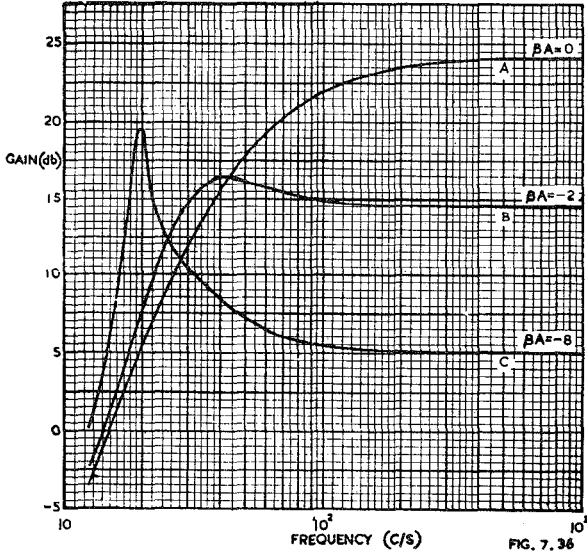

Fig. 7.36. *Frequency response of amplifier of Fig. 7.35 with zero feedback and two selected values of feedback.*

For the method of plotting this curve and for its significance see Sect. 3 of this Chapter. It should be noted that once the βA curve has been obtained the effects of feedback on the gain and phase shift of the amplifier can readily be obtained graphically as discussed below. Lines 3 to 9 of Table 2 are the mathematical equivalents of the measurements mentioned below (1 and 5) which give the degree of degeneration or regeneration and the phase shift with feedback between output and input voltages at a given frequency.
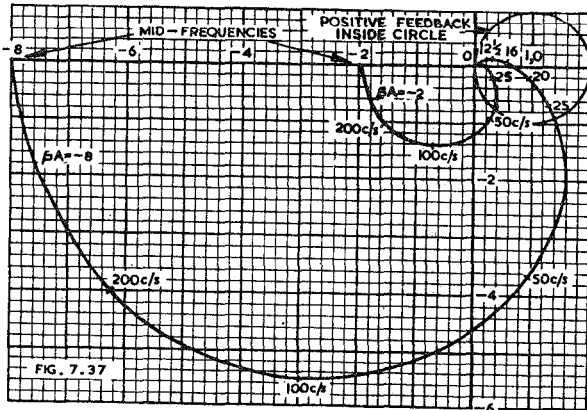

Fig. 7.37. *Polar diagram for two selected values of feedback.*

TABLE 1.

	Mid-frequency	200 c/s	100 c/s	50 c/s	37½ c/s	25 c/s	20 c/s	18¾ c/s	16 c/s	12½ c/s
(1) Coupling 1 Attenuation (db)	0∠0°	0∠4°	0.10∠7°	0.3∠14°	0.5∠18°	1.0∠27°	1.4∠32°	1.5∠34°	2.1∠38°	3∠45°
(2) Coupling 2 or 3 Attenuation (db)	0∠0°	0.3∠14°	1.0∠27°	3.0∠45°	4.5∠53°	7.0∠63°	8.5∠68°	9.1∠69.5°	10.3∠72°	12.3∠76°
(3) Couplings 1 + 2 + 3. Attenuation (db)	0∠0°	0.6∠32°	2.1∠61°	6.3∠104°	9.5∠124°	15.0∠153°	18.4∠168°	19.7∠173°	22.7∠182°	27.6∠197°
(4) Gain $A = A_1 \times A_2 \times A_3$ (db)	24.1∠0°	23.5∠32°	22.0∠61°	17.8∠104°	14.6∠124°	9.1∠153°	5.7∠168°	4.4∠173°	1.4∠182°	−3.5∠197°
(5) Gain A	16∠0°	14.9∠32°	12.6∠61°	7.7∠104°	5.3∠124°	2.9∠153°	1.9∠168°	1.66∠173°	1.17∠182°	0.67∠197°

Lines 3 and 4 are the values for the sine and cosine of the angle of phase shift [see Chapter 38 Table 72 for magnitude; Chapter 6, Sect. 3(iii) for sign]. Line 5 expresses $1 - \beta A$ in the form $a + jb$. Multiplying the magnitude of βA in line 2 by the sine of the associated angle gives the j term and multiplying by the cosine and adding 1 (because of the 1 in $1 - \beta A$) gives the real term. For example, at 100 c/s, $1 + 6.3 \times 0.4848 = 4.05$ and $6.3 \times 0.8746 = 5.5$ so that $1 - \beta A = 4.05 + j\,5.5$.

Lines 6 and 7 convert $1 - \beta A$ to polar co-ordinates again (the change to rectangular co-ordinates was necessary to add 1 to βA), line 6 being expressed in the form of the square root of the sum of the squares of the two terms in line 5 at the angle whose tangent is the imaginary term divided by the real term, e.g. at 100 c/s the angle is $\tan^{-1} 5.5/4.05 = \tan^{-1} 1.36 = 54°$ to the nearest degree. Where negative terms occur the appropriate quadrant is determined from the knowledge of the sign of the sine, cosine and tangent, all of which are known at this stage.

Line 8 is the reciprocal of line 7, so that magnitudes are divided into unity, and angles are reversed in sign. In line 9 the magnitudes are expressed in db, and in line 10 the original gain A at each frequency is multiplied by $1/(1 - \beta A)$ to give the response of the amplifier with feedback. Since lines 9 and 4 are both expressed in db, the multiplication is carried out by adding the values in decibels. Line 10 is plotted in curve C of Fig. 7.36.

Table 3 gives similar calculations when $\beta = -1/8$ and the result is plotted in curve B of Fig. 7.36, while the smaller βA polar diagram is also plotted in Fig. 7.37.

Several interesting aspects which are common to all feedback amplifiers are brought out by these calculations and curves.

TABLE 2

	Mid-frequency	200 c/s	100 c/s	50 c/s	25 c/s	20 c/s	18¼ c/s	16 c/s	12½ c/s
(1) Couplings 1+2+3. Gain	1.0	0.93	0.79	0.48	0.18	0.12	0.104	0.073	0.042
(2) $\beta A(\beta \approx -\frac{1}{2})$	$-8\angle 0°$	$-7.45\angle 32°$	$-6.3\angle 61°$	$-3.8\angle 104°$	$-1.44\angle 153°$	$-0.96\angle 168°$	$-0.83\angle 173°$	$-0.58\angle 182°$	$-0.34\angle 197°$
(3) Sine	0	0.5299	0.8746	0.9703	0.4540	0.2079	0.1219	-0.0349	-0.2924
(4) Cosine	1.0	0.8480	0.4848	-0.2419	-0.8910	-0.9781	-0.9925	-0.9994	-0.9563
(5) $1-\beta A$	$9+j0$	$7.3+j3.9$	$4.05+j5.5$	$0.08+j3.7$	$-0.28+j0.65$	$0.06+j0.20$	$0.18+j0.1$	$0.42-j0.02$	$0.67-j0.1$
(6) $1-\beta A$	$\sqrt{9^2+0^2}\angle 0°$	$\sqrt{53.3+15.2}\angle 28°$	$\sqrt{16.4+30.3}\angle 54°$		$\sqrt{0.078+0.42}\angle 113°$	$\sqrt{0.004+0.04}\angle 73°$			
(7) $1-\beta A$	$9\angle 0°$	$8.3\angle 28°$	$6.8\angle 54°$	$3.7\angle 87°$	$0.71\angle 113°$	$0.21\angle 73°$	$0.21\angle 29°$	$0.42\angle -3°$	$0.67\angle -8°$
(8) $\dfrac{1}{1-\beta A}$	$0.11\angle 0°$	$0.12\angle -28°$	$0.147\angle -54°$	$0.27\angle -87°$	$1.4\angle -113°$	$4.76\angle -73°$	$4.76\angle -29°$	$2.4\angle 3°$	$1.5\angle 8°$
(9) $\dfrac{1}{1-\beta A}$(db)	$-19.2\angle 0°$	$-18.4\angle -28°$	$-16.6\angle -54°$	$-11.4\angle -87°$	$+3.0\angle -113°$	$13.6\angle -73°$	$13.6\angle -29°$	$7.6\angle 3°$	$3.6\angle 8°$
(10) $\dfrac{A}{1-\beta A}$(db)	$4.9\angle 0°$	$5.1\angle 4°$	$5.4\angle 7°$	$6.4\angle 17°$	$12.1\angle 40°$	$19.3\angle 95°$	$18\angle 144°$	$9.0\angle 185°$	$0.1\angle 205°$

TABLE 3

	Mid-frequency	200 c/s	100 c/s	50 c/s	37½ c/s	25 c/s	20 c/s	16 c/s	12½ c/s
(1) 1+2+3 Gain	$3\angle 0°$	$0.93\angle 32°$	$0.79\angle 61°$	$0.48\angle 104°$	$0.33\angle 124°$	$0.18\angle 153°$	$0.12\angle 168°$	$0.073\angle 182°$	$0.042\angle 197°$
(2) βA ; $\beta=-\frac{1}{3}$	$-2\angle 0°$	$-1.86\angle 32°$	$-1.58\angle 61°$	$-0.96\angle 104°$	$-0.66\angle 124°$	$-0.36\angle 153°$	$-0.24\angle 168°$	$-0.146\angle 182°$	$-0.082\angle 197°$
(3) Sine	0	0.5299	0.8746	0.9703	0.8290	0.4540	0.2079	−0.0349	−0.2924
(4) Cosine	1	0.8480	0.4848	−0.2419	−0.5592	−0.8910	−0.9781	−0.9994	−0.9563
(5) $1-\beta A$	$3+j0$	$2.58+j0.98$	$1.77+j1.4$	$0.77+j0.93$	$0.63+j0.55$	$0.68+j0.16$	$0.755+j0.05$	$0.855-j0.005$	$0.92-j0.024$
(6) $1-\beta A$	$\sqrt{3^2+0^2}\angle 0°$	$\sqrt{6.65+0.96}\angle 21°$	$\sqrt{3.13+1.96}\angle 38°$	$\sqrt{0.59+0.86}\angle 50°$	$\sqrt{0.40+0.30}\angle 41°$	$\sqrt{0.46+0.02}\angle 13°$			
(7) $1-\beta A$	$3\angle 0°$	$2.76\angle 21°$	$2.26\angle 38°$	$1.21\angle 50°$	$0.84\angle 41°$	$0.69\angle 13°$	$0.76\angle 4°$	$0.86\angle 0°$	$0.92\angle -2°$
(8) $\dfrac{1}{1-\beta A}$	$0.33\angle 0°$	$0.36\angle -21°$	$0.44\angle -38°$	$0.82\angle -50°$	$1.19\angle -41°$	$1.45\angle -13°$	$1.3\angle -4°$	$1.16\angle 0°$	$1.09\angle 2°$
(9) $\dfrac{1}{1-\beta A}$ (db)	$-9.6\angle 0°$	$-8.9\angle -21°$	$-7.1\angle -38°$	$-1.7\angle -50°$	$1.6\angle -41°$	$3.2\angle -13°$	$2.3\angle -4°$	$1.3\angle 0°$	$0.7\angle 2°$
(10) $\dfrac{A}{1-\beta A}$ (db)	$14.5\angle 0°$	$14.6\angle 11°$	$14.9\angle 23°$	$16.1\angle 54°$	$16.2\angle 85°$	$12.3\angle 140°$	$8.0\angle 164°$	$2.7\angle 182°$	$-2.8\angle 199°$

(1) The degree of degeneration (or regeneration) at any frequency marked on the polar diagram Fig. 7.37 can be obtained by measuring the distance at the appropriate frequency from the curve to the point 1,0. The same scale is used for the measurement as for plotting βA and the distance gives the amount of degeneration, e.g. the 50 c/s point on the $\beta A = -8$ curve is $3.66\beta A$ units from 1,0, so gain reduction $= 3.66$ times $= 11.3$ db (compare 11.4 db Table 2 line 9). Again at 20 c/s, distance $= 0.2$ units so gain reduction $= 0.2$ times, i.e. gain increase $= 5$ times $= 14$ db (compare 14.1 db Table 2 line 9).

It follows that at whatever frequency the βA curve crosses the circle with centre 1,0 and radius 1, the gain of the amplifier is unchanged by feedback. For example, with $\beta A = -8$, the locus cuts the circle with radius 1 unit at a frequency of 28 c/s, which is therefore the frequency at which the gain of the amplifier is unchanged by feedback.

The same result may be obtained from Fig. 7.36 where the intersection of curves $\beta A = 0$ and $\beta A = -8$ is at 28 c/s. At lower frequencies the feedback is positive.

(2) When $\beta A = -8$ the low frequency peak is relatively much larger than when $\beta A = -2$ (14 db when $\beta A = -8$ and $2\frac{1}{2}$ db when $\beta A = -2$) and it occurs at a lower frequency. The reason can be seen from the polar diagram. When βA is large the curve is increased in size so that lower frequencies on the curve come closer to the 1,0 point.

(3) When $\beta A = -8$, the angle of βA varies from $0°$ to $197°$ but the angle of $1 - \beta A$ varies from $0°$ to $113°$ to $-8°$ (Table 2). The reason can be seen from the polar diagram. Note the rapid $1 - \beta A$ phase shift near the regenerative peak and the reason from the polar diagram.

(4) From the calculations it will be seen that, close to the mid-frequency, phase shift correction is approximately equal to $1/(1 - \beta A)$ e.g. when $\beta A = -8$ at 200 c/s in Table 2, a phase shift of $32°$ is reduced to $4°$.

(5) Although the feedback is regenerative as soon as the βA curve cuts the circle with centre 1,0 and radius 1, there is phase correction until the βA curve cuts the horizontal axis (between $18\frac{3}{4}$ and 16 c/s when $\beta A = -8$).
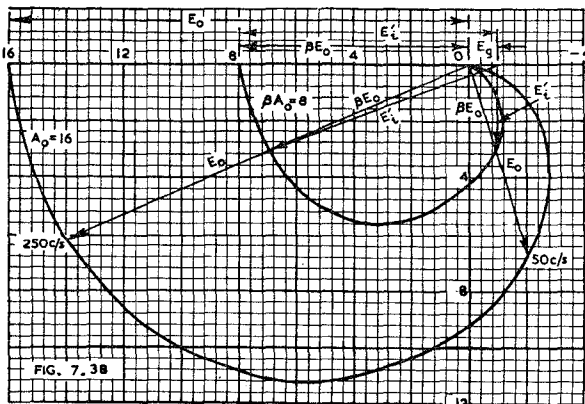

Fig. 7.38. Construction to demonstrate correction of phase shift by negative feedback.

The phase shift correction due to negative feedback is not directly obtainable from Fig. 7.37, but a construction to illustrate this effect is given in Fig. 7.38. In the diagram the larger curve ($A_0 = 16$) is the polar diagram of the gain of an amplifier, plotted as before, while the smaller curve ($\beta A_0 = 8$) is drawn for $\beta = \frac{1}{2}$. (For convenience, β is here taken as positive).

E_g represents the voltage applied between grid and cathode of the input valve, and since it is drawn 1 unit in length, the amplification curve also represents the output

voltage E_0 ($= 16$ at the mid-frequency). Similarly the βA_0 curve represents the feedback voltage, and the vector sum of the grid to cathode voltage and the feedback voltage gives the required input voltage with feedback,

i.e. $E_g + \beta E_0 = E_i'$.

Thus at the mid-frequency, 1 volt of input gives 16 volts of output without feedback, but 9 volts of input are required for the same output voltage with feedback. Since there is no phase shift, there is no phase correction.

However at 250 c/s a phase shift of 23° has occurred and E_0 has the magnitude and phase shown in the diagram. E_i' is still the vector sum of E_g and βE_0 as shown on the diagram, and the phase angle between feedback input voltage E_i' and output voltage E_0 is only about 3°.

This checks as closely as the angles can be measured with Eqn. (51) Sect. 3(viii) of this chapter, taking into account the fact that E_0 has a smaller value at 250 c/s than at the mid-frequency.

At 50 c/s a phase shift of 106° between the voltage between grid and cathode of the first valve and the output voltage is reduced to about 17° between input voltage with feedback and output voltage, which is also in agreement with eqn. (12).

As the phase shift in the amplifier proper approaches 180°, phase shift reduction from inverse feedback becomes smaller, disappearing at 180° and becoming an increase at greater angles. This can be visualized from Fig. 7.38 or demonstrated on a polar diagram on which the βA curve cuts the X axis between 0 and 1 and returns to zero after passing through angles greater than 180°.

(C) Feedback from plate, with transformer coupling

The circuit is conventional, but there is danger of instability. See Sect. 3.

(D) Multiple feedback

The theory of multiple feedback for the purpose of improving stability is covered in Sect. 3(v)E. Fig. 7.33 is an example of dual feedback, with a principal feedback loop and subsidiary feedback through the unbypassed cathode resistor in the circuit
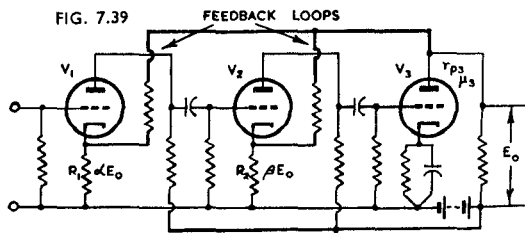

Fig. 7.39. Amplifier with interconnected positive and negative feedback loops in which the negative loop is completely contained within the positive.
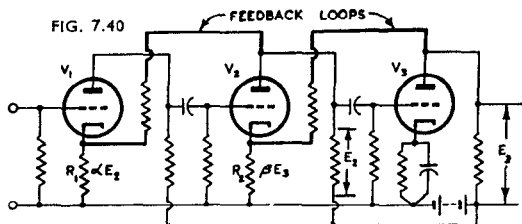

Fig. 7.40. Amplifier with interconnected feedback loops in which the loops overlap.

of V_1. In the case of a 3 stage amplifier, the unbypassed cathode resistor could be in the first or second stage. It is highly desirable, in general, for some such form of subsidiary feedback to be used. It is normally desirable that the degeneration due to the subsidiary feedback should not exceed say 10 per cent. of that due to the principal feedback loop, since the subsidiary feedback increases the distortion in the final stage. For further details and more complicated circuits see Sect. 3(v)E.

There are two general cases of interconnected feedback loops, the first in which one loop is completely contained within the other (Fig. 7.39) and the second in which the loops overlap as in Fig. 7.40.   In each of the circuits the gain of the valves V_1, V_2 and V_3 is A_1, A_2 and A_3 respectively, and the size of the resistors R is such as to introduce negligible current feedback.

For the circuit of 7.39 the gain reduction factor is

$$\frac{1}{1 - \alpha A_1 A_2 A_3 - \beta A_2 A_3}$$

(with α subject to the same convention as β) and the effective plate resistance of V_3 becomes

$$r_{p3}' = \frac{r_{p3}}{1 - \alpha A_1 A_2 \mu_3 - \beta A_2 \mu_3}$$

For the circuit of 7.40 the gain reduction factor is

$$\frac{1}{1 - \alpha A_1 A_2 - \beta A_2 A_3}$$

and the effective plate resistance of V_3 becomes

$$\frac{r_{p3}}{1 - \dfrac{\beta A_2 \mu_3}{1 - \alpha A_1 A_2}}.$$

(vii) Voltage feedback over three stages

When the feedback is taken over three stages, there is a strong likelihood of experiencing instability at extremely low and high audio (or ultrasonic) frequencies (see Sect. 3).   There are several popular circuits :
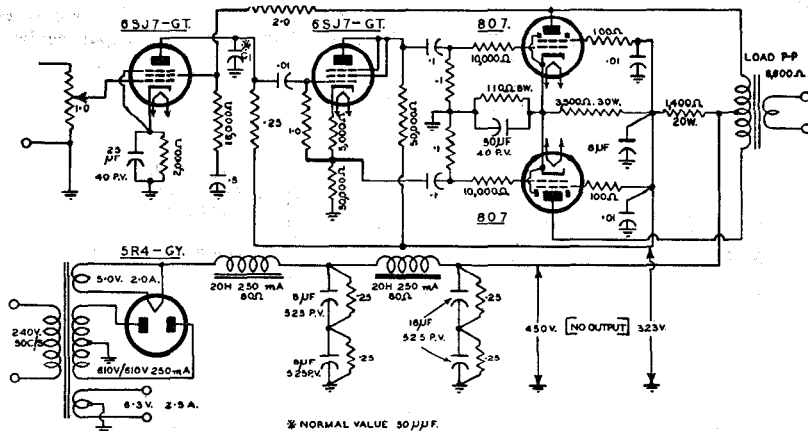
FIG. 7.42

Fig. 7.42.   30 watt push-pull amplifier with voltage feedback from one output plate to the screen of the first stage. Stability is ensured by adjustment of condenser C_1.

(A) Feedback from plate of V_3 to screen of V_1 (Fig. 7.42)

This circuit is very satisfactory for push-pull amplifiers.   As the feedback is taken from one plate only, the output transformer should have very tight coupling between the two halves of the primary (i.e. very low leakage reactance).   The amplifier and feedback network have been designed to give a flat response over a wide band of frequencies, with the intention of introducing additional attenuation at one point only. No instability has been experienced, and practically no rise in response, at low audio frequencies.   On a resistive load, the high frequency response has a slight peak at about 40 000 c/s, while on a loudspeaker load the peak is much larger and occurs at a lower frequency.   Attenuation is provided by the condenser C_1 from the plate of

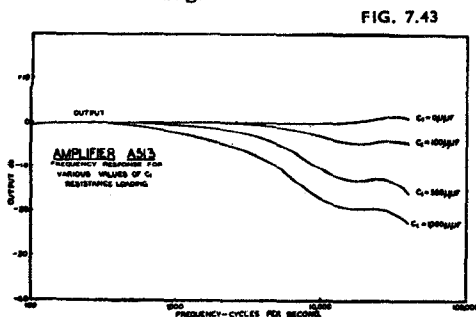the first valve to earth, with the results on frequency response shown in Fig. 7.43, for resistive loading.

FIG. 7.43

Fig. 7.43. Frequency response curves for amplifier of Fig. 7.42 on resistive load with several values of C_1.

In the absence of C_1 and on a loudspeaker load, parasitic oscillations may occur at a fairly high output level. These are largely independent of the input signal frequency but are found to occupy a small part of the cycle, with a parasitic frequency in the region of 30 000 to 40 000 c/s. The effect on music is not very apparent, but a single input frequency shows a slight " breaking up " effect when the parasitics appear. The best check is by the use of a C.R.O.

The feedback is equivalent to overall voltage feedback, and follows the usual formulae, except that it is necessary to allow for μ_{g1g2} of the first valve in the calculations for β and A. In this case :

$\beta = 16\,000/(16\,000 + 2\,000\,000) = 0.008$

A = amplification from first grid to final plate divided by μ_{g1g2}

$\quad = 2730/20 = 136.5$

Calculated gain reduction 1/2.1

Measured gain reduction 1/2.2

Input voltage for 30 watts output = 0.2 volt r.m.s.

Distortion at 30 watts output : $H_2 = 0.7\%$, $H_3 = 0.9\%$, $H_4 = 0.1\%$, $H_5 = 0.2\%$. Total 1.16% r.m.s.

Intermodulation distortion (Amplifier A513) :

Conditions of test—input frequencies 60 c/s and 2000 c/s (voltage ratio 4:1)

Power output (r.m.s.)*	4	10	14	20	24 watts
Equivalent power**	5.9	14.7	20.6	29.4	35.4 watts
Intermodulation	2.9	7.7	10	17.5	42%

*r.m.s. sum of two output frequencies

**r.m.s. sum $\times$ 25/17 to give the single frequency power having the same peak voltage swing (see Chapter 14 Sect. 3).

Any further increase in the degree of feedback is likely to prove difficult (see Section 3).

When a circuit is used which applies **negative feedback to the screen of a pentode**, the input capacitance is increased by

$\quad \mu_t C_{g1g2} |\beta| A$

where μ_t is the " triode amplification factor "

$\quad C_{g1g2}$ is the capacitance from control grid to screen

$\quad |\beta|$ is numerical value of β

and A is voltage gain from grid to the point from which the feedback is returned.

(B) Feedback from secondary of output transformer

This introduces additional phase shift in the amplifier and requires very careful design of the output transformer for all but small degrees of feedback. The feedback voltage is usually taken to the cathode of the first valve, but if there is transformer input to the first stage then it may be returned to the transformer secondary (as Fig. 7.26). This method may be applied equally to both single-ended and push-pull amplifiers.

Fig. 7.44 is the circuit of such an amplifier using push-pull triodes, with extraordinarily low distortion. It is based on the design of Williamson (Refs. F4, F6). The first triode (V_1) is direct-coupled to the grid of a phase splitter (V_2) which in turn is r.c. coupled to push-pull triodes (V_3 and V_4) and thence to the output stage. No by-pass condensers are used and the only reactances to cause phase shift at low frequencies are the grid coupling condensers in two stages. The circuit has a number of refinements which are described in the references.
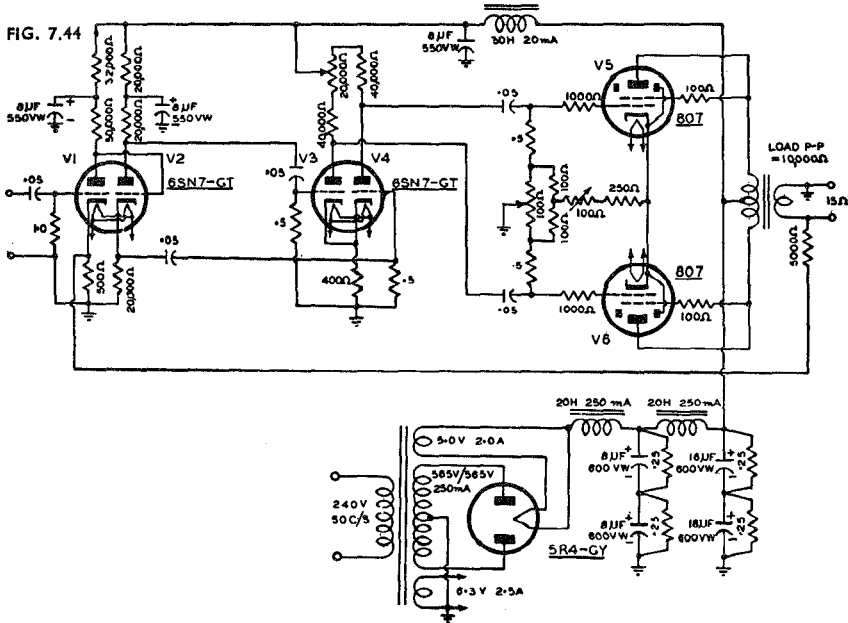

Fig. 7.44. *Amplifier employing voltage feedback from the secondary of the output transformer, with push-pull triodes. The amplifier is virtually distortionless up to an output of 11 watts, and has a smooth overload up to 16 watts.*

The original version used type L63 valves in place of each half of the 6SN7-GT, and type KT66 as triodes in the output stage. The circuit shown (A515) gives almost identical results, but type 807 valves are used in place of type KT66.

The specifications laid down by Williamson for the **output transformer** are as follows :

Primary load impedance	10 000 ohms, centre-tapped
Primary inductance	100 henrys, min.
Leakage inductance, series	
(whole prim. to whole sec.)	30 millihenrys, max.
Primary resistance	250 ohms, approx.

Commercial transformers wound to the original winding data supplied by Williamson perform quite satisfactorily, although some difficulty may be experienced where the winding structure differs from that recommended. This is brought about by the increase in interwinding capacitances which may result.

Several manufacturers have recently marketed some wide range output transformers which are a considerable improvement on that specified above.

In one particular transformer, the Partridge CFB, the series leakage inductance, the winding resistances and weight have been reduced by a third, but the power handling capabilities have been increased four times. The core distortion at a level

of 16 watts has been reduced ten times and the important leakage inductance existing between the halves of the primary has been decreased thirty times.

These improvements mean that this transformer, when used in the original Williamson circuit, will give a greater stability margin or alternatively will permit the use of a greater amount of feedback.

Distortion will be appreciably reduced in any amplifier by the use of a well designed transformer of such a type.

Distortion with amplifier A515

Power Output	H_2	H_3	H_4	H_5	Total r.m.s.
11 watts	0.01	0.04	0.01	0.015	0.045%
14 watts	0.075	0.35	0.01	0.26	0.44%
16 watts	0.04	0.92	0.11	0.45	(overload)

The smooth overload is a particularly valuable feature.

Intermodulation distortion

Conditions of test—input frequencies 60 c/s and 2000 c/s ; higher frequency 12 db lower than 60 c/s level.

Power output (r.m.s.)*	4	6	8	10	12 watts
Equivalent power**	5.9	8.8	11.8	14.7	17.6 watts
Intermodulation	0.17	0.27	0.72	3.7	8.8%

*r.m.s. sum of two output frequencies

**r.m.s. sum × 25/17 to give the single frequency power having the same peak voltage swing (see Chapter 14 Sect. 3).

Note :

In the circuit of Fig. 7.44, a capacitance may be inserted in series with the 5000 ohm resistor in the feedback circuit to provide bass boosting for equalizing purposes in record reproduction ; see Chapter 15 Sect. 9(ii)B.

A modification of this circuit, which provides for bass boosting, is given in Fig. 15.58A.

A new version of the Williamson amplifier, together with pre-amplifier and tone control (Ref. F9) is given in Figs. 17.35B,C,D,E,F,G. See also Chapter 13 Refs. F4, F5, H15.

Modified forms of the Williamson amplifier are given in Refs. F6, F8 and Chapter 13 Refs. H5, H6, H8, H10, H12.

(C) Bridge circuits

Amplifiers can be designed with bridge circuits in both input and output circuits, which prevent any modification of the input and output impedances and which also eliminate any phase shift round the feedback loop caused by reactances introduced through the input or output circuits (Refs. A1, A2, A11).

(viii) Cathode-coupled phase inverters and amplifiers

(A) Cathode coupled phase inverters

The fundamental circuit is Fig. 7.45 and requires two triode units with both cathodes linked, a common cathode resistor R_k and separate plate resistors R_{L1} and R_{L2}. Three tappings are required on the battery or potential divider across the B supply. The input voltage is applied between the two grids (Ref. G1).

If both V_1 and V_2 have identical characteristics and $R_{L1} = R_{L2} = R_L$, it may be shown (Ref. C7) that

$$E_{AB} = \mu E_s R_L / (R_L + r_p) \qquad (43)$$

where E_s = signal input voltage applied between the two grids

and E_{AB} = signal output voltage between points A and B.

Similarly, where E_A and E_B are the voltages from these points to earth,

$$E_A = \tfrac{1}{2}\left(\frac{\mu E_s R_L}{R_L + r_p}\right)\left(1 + \frac{R_L + r_p}{R_L + r_p + 2(\mu + 1)R_k}\right) \qquad (44)$$

$$\text{and } E_B = -\tfrac{1}{2}\left(\frac{\mu E_s R_L}{R_L + r_p}\right)\left(1 - \frac{R_L + r_p}{R_L + r_p + 2(\mu + 1)R_k}\right) \qquad (45)$$

The two voltages thus differ by a small amount, which can be reduced by using a large value of R_k and/or by using high mu valves. If the remaining out-of-balance is still serious, it may be eliminated entirely by making R_{L1} slightly smaller than R_{L2} so as to give exact balance.
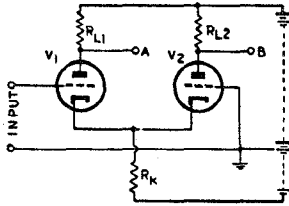

FIG. 7.45   FIG. 7.46

Fig. 7.45. *Fundamental circuit of a cathode coupled phase inverter.*
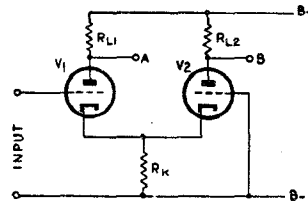*Fig.* 7.46. *Fundamental circuit of a cathode-coupled amplifier.*

One stage of a cathode-coupled phase inverter may be coupled to a second (Ref. C7) and the output of the latter will be almost exactly balanced.
The same principle may be applied to the power output stage (Ref. G6).
The circuit of Fig. 7.46 may also be used as a phase-inverter.
See also Chapter 12, Sect. 6(vi).

(B) Cathode-coupled amplifiers
The fundamental circuit of a cathode-coupled amplifier is shown in Fig. 7.46 in which V_1 and V_2 are twin triodes with a common cathode resistor R_k. The output may be taken from either A or B, A being 180° out of phase with the input, while B is in phase with the input voltage.

Output from A : Gain $A_1' = \dfrac{\mu R_{L1}}{r_p + R_{L1} + r_p R_k(\mu + 1)/[r_p + R_k(\mu + 1)]}$ (46)

(Note : R_{L2} may be shorted-out.)
The plate resistance is increased by r_p and $R_k(\mu + 1)$ in parallel. The minimum value of A' is half the gain without feedback (when $R_k = 0$). (Ref. G2).
Output from B : (Note : R_{L1} may be shorted out) (Refs. G2, G4)

Gain $A_2' = -\dfrac{\mu R_{L2}}{2r_p + R_{L2} + r_p(r_p + R_{L2})/R_k(\mu + 1)}$ (47)

If $\mu R_k \gg r_p$, then $A_2' \approx -\mu R_{L2}/(2r_p + R_{L2})$ (48)

In reality, the values of μ and r_p are not constant and not equal for the two units in the twin triode. A rigorous analysis (Ref. G17) provides a straight forward method for the accurate derivation of the amplification when the output is taken from B.

Outputs from A and B (special case of phase inverter).
The gain is somewhat less than with Fig. 7.45, but the circuit is often more convenient.
There are many applications of this circuit which are described in References (G) also C7, D6 and J2 ; see also Chapter 12 Sect. 9(iii).

(ix) Hum
The hum in a practical amplifier is not always reduced by the factor $1/(1 - \beta A)$ as in eqn. (12), Sect. 1 but sometimes is decreased according to a different law, and sometimes is even increased by the application of feedback. All hum arising from sources within the amplifier and which is independent of the signal level, may be represented by an equivalent hum voltage in series with the input voltage ; this is reduced in all cases by the factor $1/(1 - \beta A)$.

(A) **Hum originating in the grid circuit** of a single stage amplifier will be reduced by $1/(1 - \beta A)$.
(B) **Hum caused by heater-cathode leakage** is reduced by $1/(1 - \beta A)$ with

voltage feedback. With feedback from an unbypassed cathode resistor, however, the cathode-to-earth impedance is often very high, and the hum is thereby increased.

With the circuit of Fig. 7.20 the hum voltage in the output is

$$\frac{\frac{1}{2}E_h R_k}{R_k + R_{hk}} \cdot \frac{\mu R_L}{(\mu + 1)R_k + r_p + R_L} \tag{49}$$

where E_h = heater voltage (r.m.s.)
and R_{hk} = leakage resistance from heater to cathode.

(C) **Hum originating in the plate supply voltage**

[*The examples below are based on the following power amplifier valves under typical operating conditions* :

	Triode	Pentode
Type	2A3	6V6-GT
μ	4.2	215
μ *(triode connection of pentode)*		9.6
r_p	800	52 000 *ohms*
R_L	2500	5000 *ohms*
A	3.2	18.8 *times*

Voltage feedback is taken as 20% *(i.e.* $\beta = -0.2$).
Current feedback is based on $R_k = 250$ *ohms*.]

Case (1) Transformer-coupled output with voltage feedback from the secondary (Fig. 7.26)

In this case the hum is multiplied by the factor $1/(1 - \beta A)$ [= 0.61 *for type 2A3 and* 0.21 *for type* 6V6-GT]. The result is that the feedback reduces the hum slightly in the case of a triode and very considerably in the case of a pentode. If a hum-bucking coil is used, the number of turns will require readjustment for negative feedback.

Case (2) Transformer-coupled output with voltage feedback from the plate (Fig. 7.29)

In this case the feedback voltage is a fraction of the output voltage plus the hum voltage across the condenser C_2.

If V_1 is a **triode**, the hum voltage **without feedback** is

$$E_{h0} = E_h R_L/(R_L + r_p) \tag{50}$$

[*Example* : $E_{h0} = 0.76 E_h$ for type 2A3.]
while the hum voltage **with feedback** is

$$E_{h0}' = E_h R_L/(R_L + r_p') \tag{51}$$

[*Example* : $E_{h0}' = 0.85 E_h$ for type 2A3]

where E_{h0} = hum voltage output without feedback
E_{h0}' = hum voltage output with feedback
and E_h = hum voltage across condenser C_2.

Since $r_p' = r_p/(1 - \beta\mu)$ is less than r_p, the hum with feedback is greater than the hum without feedback :

$$\frac{E_{h0}'}{E_{h0}} = \frac{R_L + r_p}{R_L + r_p'} = \frac{1 - \beta\mu}{1 - \beta A} \tag{52}$$

[*Example* : $E_{h0}'/E_{h0} = 1.12$ for type 2A3]

If V_1 is a **pentode**, the hum voltage **without feedback** is

$$E_{h0} = E_h [R_L/(R_L + r_p) + R_L\mu_{g2p}/(R_L + r_p)] \tag{53}$$

[*Example* : $E_{h0} = (0.09 + 1.96) E_h = 2.05 E_h$ for type 6V6-GT]
while the hum voltage **with feedback** is

$$E_{h0}' = E_h[R_L/(R_L + r_p') + R_L\mu_{g2p}/(R_L + r_p - \beta\mu R_L)] \tag{54}$$

[*Example* : $E_{h0}' = (0.81 + 0.41)E_h = 1.22 E_h$ for type 6V6-GT]
where μ_{g2p} = mu factor from screen to plate without feedback
$\approx \mu_{g1p}/\mu_{g1g2}$ [= 215/9.6 = 22.4 *for type* 6V6-GT.]

It will be noticed that the screen and plate effects are additive ; it is only with parallel feed (see Case 3 below) that they oppose one another. The " screen " component of E_{h0}' is equal to that without feedback multiplied by $1/(1 - \beta A)$. This is not a good circuit for low hum, either with or without feedback ; these remarks also apply to Figs. 7.30, 7.31, 7.32, 7.33 and 7.42.

If the screen is perfectly filtered, the second term in eqn. (54) becomes zero. [*Example* : $E_{h0}' = 0.81E_h$ *for type* 6V6-GT.] The effect of feedback on the hum is then given by eqn. (52) with the substitution of the correct values of μ and A [*Example* : $E_{h0}'/E_{h0} = 9.2$ *for type* 6V6-GT].

Case (3) Parallel feed with voltage feedback (Fig. 7.47).

When V_1 is a triode, this is an excellent circuit for low hum, both with and without feedback, provided that L_1 has a reactance, at the hum frequency (i.e. twice supply frequency for full wave rectification) at least several times the impedance of r_p' and R_L in parallel.

When V_1 is a pentode, the hum due to the plate circuit is low but that due to the screen is high. Screen filtering (L_2C_2) is required for low total hum ; this is improved by feedback so that the screen filtering may be omitted in some applications.

Without feedback, when V_1 is a **triode**, the hum voltage is applied to a voltage divider with ωL_1 in one arm and R in the other, where $R = r_p R_L/(r_p + R_L)$. For ease in calculation it may be expressed in the form

$$E_{h0} = \cos\theta . E_h \text{ where } \theta = \tan^{-1} L_1/R \tag{55}$$

[*If* $\omega L_1 = 25\ 000$ *ohms, and* $R = 606$ *ohms as for type* 2A3, *then* $\theta = \tan^{-1} 41.2 = 88.6°$. *Thus* $\cos\theta = 0.024$ *and* $E_{h0} = 0.024E_h$].

With feedback, $E_{h0}' = E_{h0}/(1 - \beta A)$ (56)

[*Example* : $E_{h0}' = 0.024/1.64 = 0.015$ *for type* 2A3].

When V_1 is a **pentode without feedback**

$$E_{h0} \approx E_h[\cos\theta - R_L\mu_{g2p}/(R_L + r_p)] \tag{57}$$

where $\theta = \tan^{-1} \omega L_1/R$ as for the triode case
and $R = r_p R_L/(r_p + R_L)$.

The second term in eqn. (57) is only approximate, since it is assumed that the shunting effect of ωL_1 on R_L has no effect on the gain ; this is very nearly true when ωL_1 is at least equal to $10R_L$.

[*Example* : *If* $\omega L_1 = 25\ 000$ *ohms, and* $R = 4560$ *ohms as for type* 6V6-GT, *then* $\theta = \tan^{-1} 5.48 = 79.7°$. *Thus* $\cos\theta = 0.179$ *and* $E_{h0} = (0.179 - 1.96)E_h = -1.78E_h$. *With screen filtering, this would be considerably improved.*]

With feedback, $E_{h0}' = E_{h0}/(1 - \beta A)$. (58)

[*Example* : $E_{h0}' = (-1.78/4.8)E_h = -0.37E_h$ *for type* 6V6-GT.]

If L_1 is replaced by a resistor R, the necessary substitution for $j\omega L$ may be made in the equations for hum. Similarly, the transformer-coupled load (R_L) may be replaced by a following grid resistor R_g.
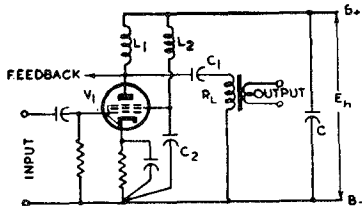

FIG. 7.47   FIG. 7.48

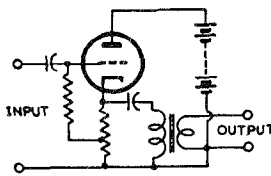*Fig. 7.47. Circuit of an amplifier with parallel-fed transformer output and negative feedback supplied from the plate.*
Fig. 7.48. Cathode follower with parallel feed.

With this type of amplifier, hum balancing arrangements inside the amplifier are unaffected by the feedback. An adjustment of the hum bucking coil (if used) in the loudspeaker will be necessary. Any type of feedback circuit may be used, either from the plate, or from the primary or secondary of the output transformer.

Case (4) Cathode follower with parallel feed (Fig. 7.48)

In this case the plate resistance is reduced by the factor $1/(\mu + 1)$, and the hum output voltage is decreased by the factor $1/(\mu + 1)$. If a pentode is used, the screen

capacitance by-pass to cathode (C in Fig. 7.16) may be adjusted to neutralize hum (Ref. C14).

Case (5) Transformer-coupled output with feedback from an unbypassed cathode resistor (Fig. 7.49)

The hum arising from the plate and screen circuits is the same as for Case (2) except that r_p' is now greater than r_p (Eqn. 51) : $r_p' = r_p + (\mu + 1)R_k$. With a triode the hum is less with current feedback than without feedback. [*Example type 2A3* : $E_{h0}' = 0.54 E_h$]. With a pentode the total hum from plate and screen is always decreased by current feedback

$$E_{h0}' = E_h \left[\frac{R_L}{R_L + r_p + R_k(\mu + 1)} + \frac{\mu_{g2p} R_L}{R_L + r_p + (\mu + \mu_{g2p})R_k} \right] \qquad (59)$$

where the first term is for the plate, and the second for the screen circuit.

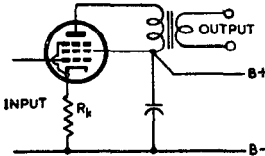[*Example* : $E_{h0}' = (0.04 + 0.97)E_h = 1.01 E_h$ *for type 6V6-GT*.]

FIG. 7.49   FIG. 7.50

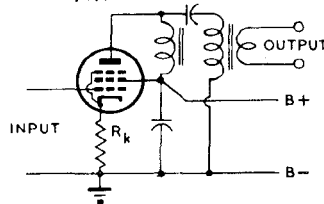*Fig.* 7.49. *Transformer-coupled output with feedback from an unbypassed cathode resistor.*

Fig. 7.50. *Parallel-fed transformer-coupled output with feedback from an unbypassed cathode resistor.*

Case (6) Parallel feed, with feedback from an unbypassed cathode resistor (Fig. 7.50)

With a pentode, the " plate " hum is increased by feedback from an unbypassed cathode resistor, while the " screen " hum is the same as for transformer-coupling (Eqn. 59) except for the sign.

$$E_{h0}' \approx E_h \left[\frac{r_p + R_k(\mu + 1)}{R_L + r_p + R_k(\mu + 1)} - \frac{\mu_{g2p} R_L}{R_L + r_p + (\mu + \mu_{g2p})R_k} \right] \qquad (60)$$

The first term also applies to a triode. The approximation is from regarding R_k as the load, instead of R_k in parallel with ωL.

[*Example of triode* : $E_{h0}' = 0.46 E_h$ *for type 2A3.*
Example of pentode : $E_{h0}' = (0.95 - 0.97)E_h = -0.02 E_h.$
This gives almost exact hum neutralization.]

References to hum : B3, B4, B5, B7.

Summary—hum originating in the plate and screen supply voltages

With triodes, the output hum voltage is always less than the hum voltage from the plate supply. In the examples given above, it varies from 46% to 85% of the plate supply hum voltage with one exception. The exception is transformer-coupled output with parallel-feed, feedback being taken from the plate, for which the output hum is only 1.5% of the plate supply hum voltage (Case 3).

With pentodes or beam power amplifiers the output hum may exceed the plate supply hum voltage, owing to the effect of the screen. This occurs with Case (2) which is the conventional transformer-coupled output both without feedback and with voltage feedback from the plate. The only circuit giving very low output hum is Case (6) which applies to transformer-coupled output with parallel feed and with feedback from an unbypassed cathode resistor. The second best is Case (1) with feedback from the secondary, while the third best is Case (3) with parallel feed, having feedback from the plate.

The examples given above have been tabulated for ease in comparison, including some cases also without feedback, and some with the screen circuit perfectly filtered.

Case	Fig.	Relative hum voltage with		Conditions
		2A3	6V6-GT	
1.	7.26	0.61	0.21	with feedback
2.	7.29	0.76	2.05	without feedback
		0.85	1.22	with feedback
		—	0.09	{ without feedback and with screen perfectly filtered
		—	0.81	{ with feedback and with screen perfectly filtered
3.	7.47	0.024*	1.78	without feedback
		0.015*	0.37	with feedback
		—	0.179	{ without feedback but with screen perfectly filtered
		—	0.037	{ with feedback and with screen perfectly filtered
5.	7.49	0.54	1.01	with feedback
6.	7.50	0.46	0.02*	with feedback

*Very low output hum.

(x) Some special features of feedback amplifiers

(A) It is important to ensure that the frequency range of the input signal applied to a feedback amplifier does not extend in either direction beyond the " flat " frequency range of the amplifier. The reason for this is explained below.

The voltage applied to the grid of the first valve in a multi-stage amplifier with overall feedback is equal to the difference between the input signal and the fedback voltage. For example, assuming 20 db of feedback, the signal voltage may be 1 volt, the fed-back voltage 0.9 volt and the voltage applied to the first stage will then be $1 - 0.9 = 0.1$ volt. At some frequency well outside the " flat " frequency range of the amplifier, where the amplifier output is 20 db down, the fed-back voltage will be 0.09 volt for a 1 volt signal, leaving a difference of 0.91 volt applied to the grid of the first valve—i.e. nine times its normal input. This may not overload the first stage, but it will probably overload the subsequent stages and cause serious distortion, even though the frequency is itself inaudible.

Even at a frequency which is only 1 db down, the signal voltage may be 1 volt, the fed-back voltage will then be 0.8 volt and the difference of 0.2 volt will be applied to the grid of the first valve ; twice normal. It is therefore essential to restrict the input range of frequencies, and it is also advisable to place the coupling having the highest attenuation as close as possible to the input of the amplifier (Ref. F7).

(B) **Damping on transients**

See Sect. 3(v) including " Design Tests."

(C) **Overload test**

See Sect. 3(v) " Design Tests."

(xi) Combined positive and negative feedback

The principle of combined positive and negative feedback has been outlined in Sect. 1(v).

A practical version of such an amplifier with push-pull output is shown in Fig. 7.51 (Ref. F10). The overall negative feedback is obtained from the secondary of the output transformer T_1 and is fed through R_5 to the cathode of V_2. Shunt capacitor C_5 affords some feedback phase correction at very high frequencies. The feedback gain reduction is 9 db, and becomes 11 db with the positive feedback disconnected.

The positive feedback is obtained from the grid of V_4 and is fed through R_1 and C_1 to the grid of V_2. The positive feedback voltage is developed primarily across R_2
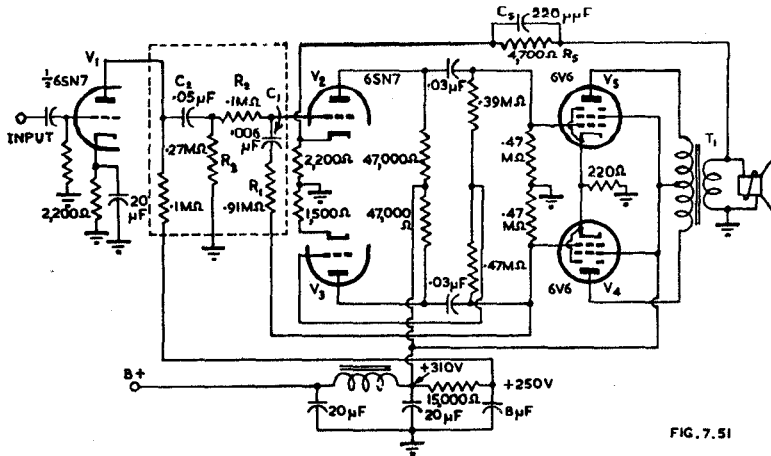
Fig. 7.51. *Two stage amplifier using combination of positive and negative feedback*
(*Ref.* F10). T_1 *ratio is* 10,000 : 15 *ohms.*

and C_2 since the plate resistance of V_1 is relatively small, and the input resistance of the grid of V_2 is high. The increase in gain due to positive feedback is about 26 db at 400 c/s. About one tenth of the voltage on the grid of V_4 is fed back to the grid of V_2. The resistance of R_1 is therefore made about nine times that of R_2, and C_2 has about nine times the capacitance of C_1. Because of the highly degenerative nature of the phase inverter, the balance is not appreciably affected by the additional load of the positive feedback network.

Some phase shift in the positive feedback is obtained at extreme frequencies in the stages V_2 and V_3 due to electrode and stray capacitances, and due to the blocking capacitors. The input capacitance of the grid of V_2 causes a further phase shift, so that the polarity of the product $\beta_1 A_1$ reverses from positive to negative at extremely high frequencies, where β_1 and A_1 apply to stage V_2. The input capacitance of V_2 is primarily Miller Effect due to feedback through its grid-plate capacitance at very high frequencies where the overall feedback is positive or small.

In some designs it may be necessary to connect a small capacitor from the grid of V_2 to earth, or to use a more elaborate phase shift network to obtain a sufficiently rapid phase turnover in the local feedback.

At extremely low frequencies most of the local feedback current flows through R_3 instead of through C_2, so that a phase shift is obtained, which together with the phase-shifting action of the $0.03\mu F$ blocking capacitors in stages V_2 and V_3, is sufficient to cause the desired phase reversal. In practice, the phase reversal frequencies are placed as far outside the desired pass band as good stability permits.

The output transformer is quite small, the core area being only ¾ in. by ¾ in.

The maximum third harmonic distortion at 400 c/s is 0.24%, higher harmonics being relatively small, at 8 watts output, measured at the secondary. At 100 c/s the highest harmonic is the second, with 0.12% for 8 watts output. At 50 c/s, with 5 watts output, third harmonic distortion is 0.88%, while at 2000 c/s with 4 watts output the third harmonic is 0.23%. The intermodulation distortion is 40% with no feedback, 8% with negative feedback alone and 1.9% with combined positive and negative feedback under the following conditions—output 8 watts, 4:1 ratio with frequencies 60 and 7000 c/s. With frequencies of 100 and 7000 c/s the intermodulation distortion is only 0.84%. The output circuit regulation is quoted as 0.1 db at 400 c/s as compared with 2.7 db for positive feedback disconnected, or 19 db with no feedback (Ref. F10).

It is possible to apply overall negative feedback together with positive feedback from a tapping on the cathode resistor of a phase-splitter through a coupling resistor to the unbypassed cathode of the preceding r.c. pentode, and thus increase the gain. The increased gain may require additional filtering (Ref. F16). This may also be used with direct coupling (Fig. 7.51A and Ref. F11).
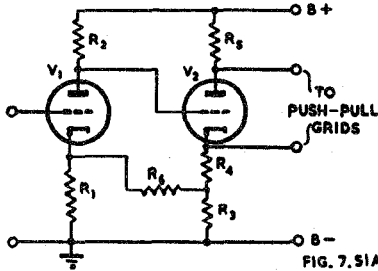

Fig. 7.51A. *Direct-coupled triode amplifier V_1 and phase splitter V_2, with positive feedback from the cathode circuit of V_2 to the cathode of V_1 (Ref. F11).*

It is possible to omit the by-pass condensers from the cathode bias resistors in the final and penultimate stages of an amplifier or receiver (or any other two successive stages) and to bring the gain back to normal by means of positive feedback, using only one resistor coupling the two cathodes as in Fig. 7.51B.

Fig. 7.51B. *Positive feedback from the cathode of V_2 through R_4 to the cathode of V_1 to offset the loss of gain by the omission of both by-pass condensers.*
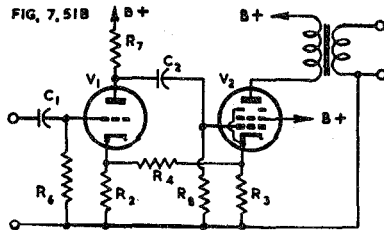
This circuit (Fig. 7.51B) may also be used with any desired degree of feedback. The effective plate resistance of V_2 may be increased or decreased by the positive feedback as shown by the following analysis and Fig. 7.51C. Here R_1 replaces R_7 and R_8 in parallel, no input signal is applied but a signal generator E is inserted in the plate circuit. It is assumed that R_4 is very much greater than R_3. We thus obtain

$$ I = \frac{E}{R_A} - \frac{IR_3\mu_2}{R_A} + \frac{IR_3|\beta|A_1\mu_2}{R_B} \tag{61} $$

where $R_A = R_5 + r_{p2} + R_3$

$R_B = R_5 + r_{p2} + R_3(\mu_2 + 1)$

$$ A_1 = \frac{\mu_1 R_1}{R_1 + r_{p1} + R_2} $$

and $\quad |\beta| = \dfrac{R_2}{R_2 + R_4}$.

From (61) we may derive

$$ I = \frac{E}{R_B - R_3|\beta|A_1\mu_2 R_A/R_B} $$

$$ = \frac{E}{R_5 + r_{p2} + R_3[\mu_2 + 1 - (|\beta|A_1\mu_2 R_A)/R_B]} $$

and hence $\quad r_{p2}' = r_{p2} + R_3[\mu_2 - (|\beta|A_1\mu_2 R_A)/R_B] \tag{62}$

where $|\beta|$ and A have the values defined below eqn. (61). The second term on the right hand side of eqn. (62) is the increase in effective plate resistance due to the unbypassed cathode resistor of V_2, while the third term is the decrease in effective plate resistance due to positive feedback. It is obvious that, by a suitable choice of

βA_1, it is possible to obtain any desired value of effective plate resistance from very high values to very low, even zero or negative values. If the resistor R_4 is made variable, the effective plate resistance may be varied over a wide range. The addition of overall negative feedback will make the gain nearly constant for all settings of R_4, and will also decrease the distortion and reduce the effective plate resistance to any values desired (Ref. F17). See also Ref. J19.

The addition of positive feedback will increase the phase shift and may increase the tendency towards instability at very low and very high frequencies ; care should be taken to reduce to a minimum the phase shift within the positive feedback loop. The general principles of designing for stability are the same as for negative feedback —see Sect. 3.

It is normally inadvisable to return the positive and negative feedback loops to the same point in the input circuit. If both are taken to the same electrode, a bridge network may be used for isolating their effects. Each 6 db increase in gain due to positive feedback will double the distortion in this stage ; for example 24 db increase in gain will increase the distortion in this stage by 16 times. This will then be reduced by the negative overall feedback in accordance with the usual relationship. References to positive and negative feedback—Refs. F10, F11, F12, F13, F15, F16, F17 ; Sect. 3(v)E.
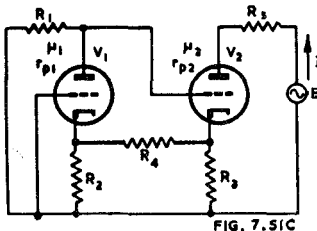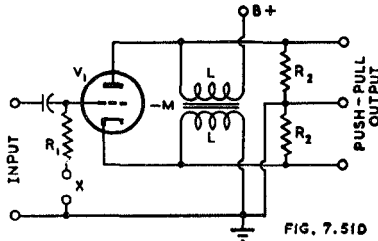

Fig. 7.51C.   Analysis of circuit of Fig. 7.51B to determine the effective plate resistance of V_2.

Fig. 7.51D.   Choke coupled phase inverter (Ref. F14).

(xii) Choke-coupled phase inverter (Ref. F14)

The choke-coupled phase inverter of Fig. 7.51D has been designed to give higher output voltages than the conventional phase splitter, to be practically unaffected by the flow of grid current in V_1 and to have small phase shift. The two identical chokes L in the plate and cathode circuits are tightly coupled so that this duty is performed by a 1 : 1 a-f transformer. This transformer applies negative voltage feedback to the input circuit, and thus causes a low effective plate resistance. Design must aim at a high total inductance to keep phase errors small, and a low leakage inductance if it is desired to keep the effect of grid current small. A fixed bias voltage is applied between terminals X, having polarity and magnitude to give, in conjunction with the voltage drop in the cathode choke, the correct grid bias. The complete analysis is given in Ref. F14.

The voltage gain is given approximately by

$$A' \approx \frac{\mu R_2}{(\mu + 2)R_2 + r_p} \text{ where } X \gg R_2$$

and the phase shift between input and output is given exactly by

$$\tan \phi = \frac{r_p}{(\mu + 2)R_2 + r_p} \cdot \frac{R}{X}$$

where $X = 2\omega(L + M)$.

X may be taken to include any self-capacitance or other stray capacitance.

As a result of the small phase shift, this circuit may readily be used in feedback amplifiers.

SECTION 3 : STABILITY, PHASE SHIFT AND FREQUENCY RESPONSE

(i) Stability and instability (ii) Conditions for stability (iii) Relationship between phase shift and attenuation (iv) Design of 1 and 2 stage amplifiers (v) Design of multi-stage amplifiers (vi) Effect of feedback on frequency response (vii) Design of amplifiers with flat frequency response (viii) Constancy of characteristics with feedback (ix) Effect of feedback on phase shift.

(i) Stability and instability

Equation (2b) of Sect. 1 gives the amplification with negative feedback, at the mid-frequency, in the form

$A' = A/(1 - \beta A)$.

It is obvious that both A and βA must be vector quantities having different magnitudes and phase angles at very low and very high frequencies from those they have at the mid-frequency. If βA is real and negative, we have negative feedback with a decrease in amplification. If βA is real, positive, and less than 1 in absolute value, we have positive feedback with an increase in amplification. If $\beta A = 1$ the amplification becomes theoretically infinite, and the system is unstable. If βA is real, positive, and greater than 1 in absolute value, the system will be only " conditionally stable " and is likely to oscillate during the warming-up period.

The quantity βA is called the **loop amplification.**

The conditions for stability described in (ii) below are based on the well-known Nyquist criterion of stability. There is, however, an alternative known as the Routh-Hurwitz stability criterion which is useful when the expression is in analytical form. See Ref. H16.

(ii) Conditions for stability

Before proceeding with any particular amplifier, it is necessary to calculate or measure its amplification and phase angle over a very wide band of frequencies. The frequency and phase angle characteristics of an imaginary amplifier are shown in Fig. 7.52. The frequency f_0 at which the phase angle is zero is called the mid-frequency ; f_1 and f_2 are the frequencies at which the phase angle displacement is a lead and lag of $90°$ respectively ; frequencies f_3 and f_4 are those giving a lead and lag of $180°$ respectively.

Taking first the case of a single stage with r.c. coupling, it is possible to plot the locus of the values of βA from zero frequency to infinity with polar coordinates (Fig. 7.53A). This is commonly known as a " Nyquist diagram." If A_0 is the amplification at the mid-frequency f_0, this will normally be its maximum value and the phase angle displacement will be zero. It will, of course, be negative for negative feedback, thus giving the vector βA_0 as drawn. At any lower frequency f_1, the value of A may be taken from the curve for this amplifier ; let us call it A_1. The value of β may be regarded as a fixed negative fraction less than unity. We thus have the value of βA_1, and the phase angle displacement ϕ_1 may also be determined from the phase angle curve. This may then be plotted as a radial vector from 0 to βA_1, with a leading phase angle displacement ϕ_1, which implies that the phase of the feedback voltage leads the input voltage by the angle ϕ_1.

Similarly for any higher frequency f_2 ; we can plot βA_2 with a length proportional to the amplification at this frequency, and a lagging phase angle displacement ϕ_2. As the frequency increases still further, we have a smaller βA_3 with a larger phase displacement, while in the extreme limit $\beta A = 0$ with $\phi = 90°$ lagging. At the other frequency extreme ($f = 0$) we have $\beta A = 0$ with $\phi = 90°$ leading. In this case the shape of the locus is a circle which is all in the negative region ; this amplifier is therefore always stable.

Any amplifier may have its βA locus plotted by this method ; a typical case is Fig. 7.53B for a 2 stage r.c.c. amplifier. Here βA_1 still has appreciable length with ϕ_1 greater than $90°$, but $\beta A = 0$ when $\phi = 180°$ either leading or lagging.
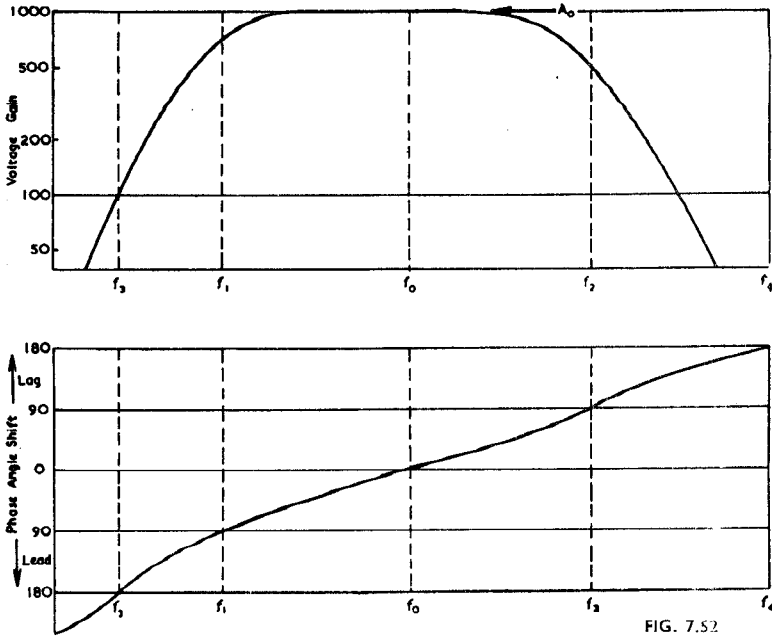
Fig. 7.52. Voltage gain and phase angle shift of an imaginary amplifier. The mid-frequency is f_o and the maximum voltage gain A_o.

The distance from any point on the locus to point K is equal to $(1 - \beta A)$ where β is negative for negative feedback (see Fig. 7.53B).

If it is desired to find the magnitude and phase angle of βA at any frequency, it is only necessary to measure the gain without and with feedback, to calculate the ratio A_0/A' between them, to draw a circle with centre K and radius A_0/A' units, then to draw a second circle with centre 0 and radius equal to the measured fed-back voltage $|\beta A|$. The line from the origin to the point of intersection between the two circles is the complex number βA.

For any amplifier to be stable, the βA locus must not include the point K (1,0). It is also desirable for the locus not to cut the X axis beyond the point K,
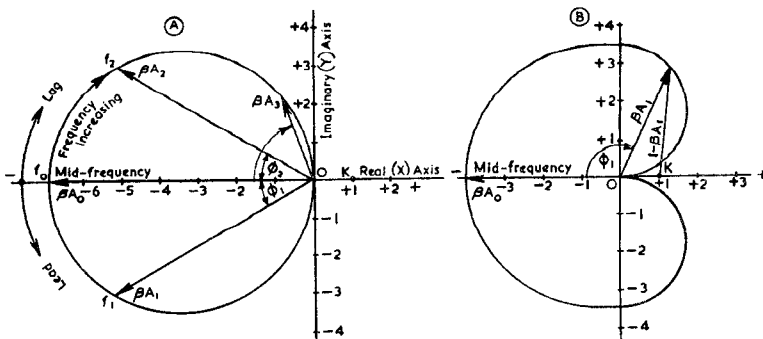

FIG. 7.53

Fig. 7.53. (A) Locus of βA vectors (" Nyquist diagram ") for a single stage resistance capacitance coupled triode amplifier with fixed bias (B) Similar locus for a 2 stage amplifier.

otherwise oscillation may occur during the warming-up period ; such an amplifier is said to be conditionally stable.   In the case of more complicated circuits, particularly those with peculiarly shaped βA loci, it is advisable to plot loci for successively increasing values of β up to and somewhat beyond the desired value of β.   The criterion of stability should be applied to each—that the locus should not enclose the point K (1,0).

The following rule is also helpful as it covers most cases and avoids having to draw a βA locus. **If βA is less than unity at $\phi = +180°$ and at $\phi = -180°$ the amplifier is stable.**

Another useful rule is that **the maximum permissible value of βA is equal to the maximum value of A (i.e. A_0) divided by the value of A at $\phi = \pm 180°$ (whichever is the larger).**

Also, **if the phase shift is not more than 180° at zero and at infinite frequencies, the amplifier is always stable with any value of β.**

The " attenuation " is the attenuation with respect to the maximum value of A (i.e. A_0).   Both the attenuation and the loop amplification βA may be expressed in decibels.
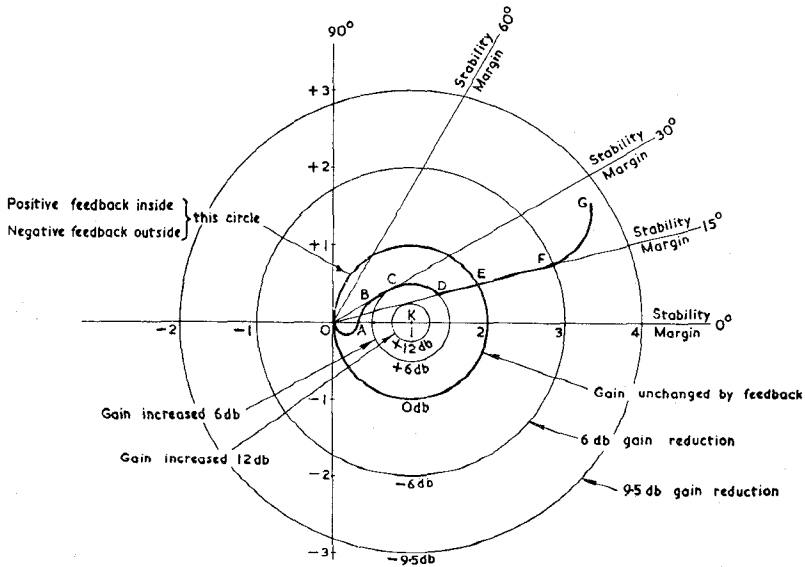

FIG. 7.54

Fig. 7.54.   Additional information furnished by a " Nyquist diagram " (see also Fig. 7.53 A and B.)

Certain aspects of the polar diagram for amplifiers with feedback are shown in Fig. 7.54.   Circles are shown with centres at point K (1,0) with radii of 0.25, 0.5, 1.0, 2 and 3 units.   The circumference of the circle with a radius of 1 unit indicates the locus of points at which the gain is unchanged by feedback.   At all points outside this circle, the feedback is negative and degenerative.   For example, the circumference of the circle with a radius of 2 units is the locus of points having a gain reduction of 6 db.   At all points inside the circumference of the circle with a radius of 1 unit, the feedback is positive and regenerative although not necessarily unstable. For example, the circumference of the circle with a radius of 0.5 unit is the locus of points having a gain increase of 6 db.

Instability occurs when the point K is included by the βA locus.   Straight lines are drawn radiating from the origin with stability margins of 15°, 30°, 60°, and 90°. The 30° line is tangential to the circle with a radius of 0.5 unit, so that the point of

tangency has 30° stability margin and 6 db gain increase. At other points along this line the gain increase will be less than 6 db.

An example of the use of the polar diagram in connection with the design of negative feedback amplifiers is given in Sect. 2(vi)B and Figs. 7.37 and 7.38.

It would be permissible for an amplifier to have a βA locus such as the line OABCDEFG in Fig. 7.54 (see Ref. A29). Here the maximum increase of gain due to positive feedback is 6 db, which occurs between C and D. The feedback is negative over the path EFG. The distance OA should not exceed say 0.3 unit, so as to allow for a possible increase in amplifier gain of about 3 db without further increasing the peak of 6 db.

Feedback amplifiers which are designed with a small stability margin should have narrow tolerances on components that have a direct effect on gain or frequency characteristics. It is desirable for βA to be measured and its locus plotted for the pilot model of each design for three conditions, with normal (bogie) valves and with valves at the upper and lower limits for mutual conductance. This is only required in the region of the low and high frequency peaks of response, which are normally beyond the working frequency range. Alternatively and more simply, the height of the low and high frequency peaks may be measured in relation to the response without feedback ; in accordance with good practice, the rise with feedback should not exceed 6 db, and lower values are desirable.

A special stability problem occurs when an amplifier may not always be connected to its correct load. One way of overcoming this problem is to design a high-and-low pass filter pair for connection between the amplifier and the load, the high-pass filter being terminated by a resistance (Ref. A29).

If an amplifier is stable with the output terminals open-circuited, short-circuited, or operated into its rated resistive load, then it is stable under all load conditions.† The measurements of stability on open-circuit and under rated load resistance can be made with an oscilloscope across the output terminals to check for the presence of ultrasonic oscillations. A check on the short-circuit conditions can best be made using a high-frequency ammeter across the output terminals. An oscilloscope of very high input impedance might also be used by connecting it across the input of one of the stages in the feedback loop (Ref. A28).

(iii) Relationship between phase angle displacement and attenuation

The phase angle displacement of an amplifier is normally a function of the attenuation characteristic. Because of this fact, it is possible to design a feedback amplifier on the basis of either the phase angle displacement or the attenuation characteristic. If the attenuation characteristic* is a straight line with a slope of 6 decibels per octave, then the ultimate phase angle is 90° and so on in proportion ; 12 db/octave gives 180° phase angle and 18 db/octave gives 270°. The condition which must be satisfied is that the amplifier is a " minimum phase shift network." This condition is satisfied by most amplifiers, the exceptions being (1) when it contains a transmission line or equivalent circuit with distributed constants, and (2) when it includes an allpass section, either as an individual structure or in a combination which can be replaced by an all-pass filter section plus some other physical structure (Refs. H6, H10, H11).

Fig. 7.54A shows ultimate slopes of 6 and 12 db/octave and higher slopes. The 12 db/octave attenuation characteristic is the limiting value for stability, being on the verge of instability. A practical amplifier requires a safety margin between the slope of its design characteristic and the limiting value of 12 db/octave. A typical design slope is 10 db/octave, giving an angular safety margin of 30°.

Any single reactive element such as a grid coupling condenser, a shunt capacitance or a shunt inductance in conjunction with a resistance provides an attenuation characteristic with an ultimate slope of 6 db/octave. Such a combination is known as a single time constant circuit. Attenuation characteristics are given in Fig. 4.36 (grid coupling condenser or shunt inductance) and Fig. 4.38 (shunt capacitance), for a single time constant in each case.

*Plotted with logarithmic frequency scale.
†This applies to a resistive load only. With a capacitive load, instability is likely to occur.

If a parallel-fed transformer is used, the coupling condenser resonates with the primary inductance at some low frequency and the ultimate slope of the stage is approximately 12 db/octave below the frequency of resonance. If the secondary of a transformer is unloaded, the leakage inductance resonates with the distributed capacitance, giving a slope for the stage of approximately 12 db/octave above the frequency of resonance. Each of these combinations is equivalent to two time constants.
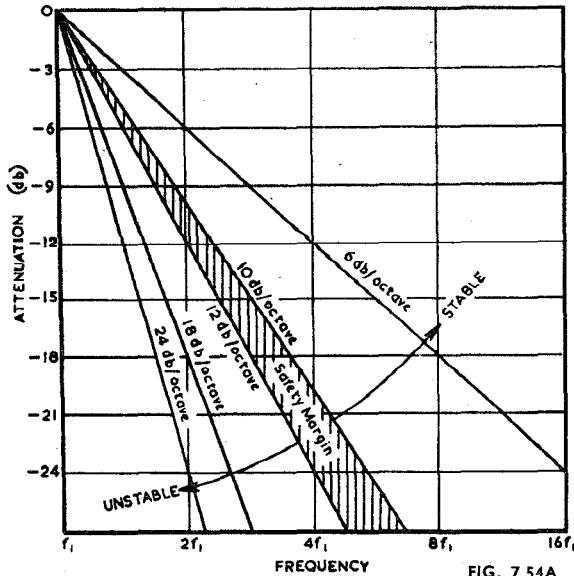

FIG. 7.54A

Fig. 7.54A. Attenuation characteristics for 6, 12, 18 and 24 decibels per octave as given by the " asymptotic " or ultimate characteristics of the amplifiers in their simplest form. The region beyond 12 db/octave is unstable ; that below 10 db octave is stable, and the region between 10 and 12 db/octave is a safety margin to allow for discrepancies between the calculated design and the finished amplifier.

Cathode and screen by-passing

In the case of cathode and screen by-passing, the attenuation characteristics are noticeably curved and the phase angle displacement cannot be calculated as for a straight line, but may be obtained from Figs. 12.3C and 12.11B respectively. In both cases the phase angle displacement reaches a maximum value at some frequency and gradually decreases at lower and higher frequencies.

Cathode by-passing

The maximum slope of the attenuation characteristic and maximum phase angle displacement are given below

B	=		2	3	5	10	
Max. loss	=		6	9.5	14	20	db
Max. slope	=		2.25	2.6	3.6	4.8	db/octave
Max. angle	=		20°	30°	42°	55°	

where $B = 1 + \dfrac{R_k(\mu + 1)}{r_p + R}$ and $R = \dfrac{R_L R_{g2}}{R_L + R_{g2}}$

For a pentode $B \approx 1 + g_m R_k$.

For typical r.c. triodes and pentodes, B is usually less than 3.

Screen by-passing

This has an effect similar to that of cathode by-passing

B	=	2.5	5	10	20
Max. loss	=	8	14	20	26 db
Max. slope	=	2.4	4	4.5	5.4 db/octave
Max. angle	=	25°	42°	54.5°	64.5°

where $B = 1 + \dfrac{R_s g_m}{m\mu_t(1 + R_l/r_v)}$

R_s = series screen resistor in ohms

g_m = mutual conductance at operating point, in mhos

m = ratio of plate to screen currents

μ_t = triode amplification factor

R_L = plate load resistance in ohms

and r_v = plate resistance in ohms.

Phase angle characteristics of " step circuit "

The step circuit of Fig. 7.59A has a maximum phase angle displacement at the frequency of the half-attenuation point, and approaches zero at lower and higher frequencies. The curves for a particular case are plotted in Fig. 7.54B, and the value of the maximum phase angle as a function of the attenuation in Fig. 7.54C.
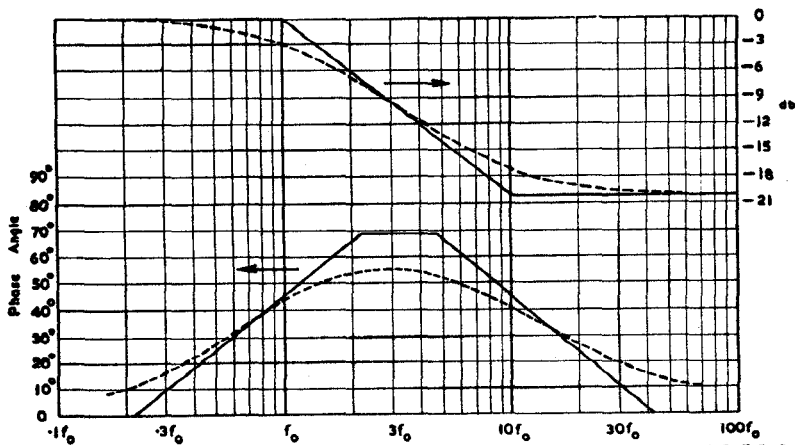

FIG. 7.54B

Fig. 7.54B. Attenuation and phase characteristics of " step circuit " Fig. 7.59A for particular case with step attenuation 20db (Ref. H18).
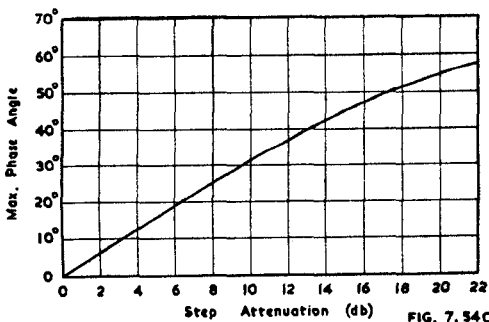

Fig. 7.54C. Maximum phase shift as a function of attenuation in the circuit of Fig. 7.59A (Ref. H18).

Step   Attenuation   (db)   FIG. 7.54C

Total slope of attenuation characteristic

The total slope of the attenuation characteristic of an amplifier in db/octave, at any frequency, is the sum of the slopes of the attenuation characteristics of all the reactive elements in the amplifier at the same frequency, provided that the slopes are constant.

In practice, the best procedure is to plot the individual attenuation characteristics with a linear db scale, then to add the decibel ordinates at various frequencies to determine the total attenuation characteristic. The slope of the latter may then be determined graphically.

Determination of phase angle

When the attenuation characteristic is a straight line forming a sharp angle at the cut-off frequency f_0, as in Fig. 7.55A, the phase angle displacement is $k \times 45°$ at f_0, where k is one sixth of the slope in db/octave, and the phase angle characteristic is asymptotic to $0°$ and $k \times 90°$.

When the attenuation characteristic is that of a normal r.c. amplifier, $k \times 3$ db down at f_0 (as broken curve in Fig. 7.55D) the phase angle displacement is also $k \times 45°$ at f_0, but at other frequencies the phase angle characteristic differs slightly from that of Fig. 7.55A.
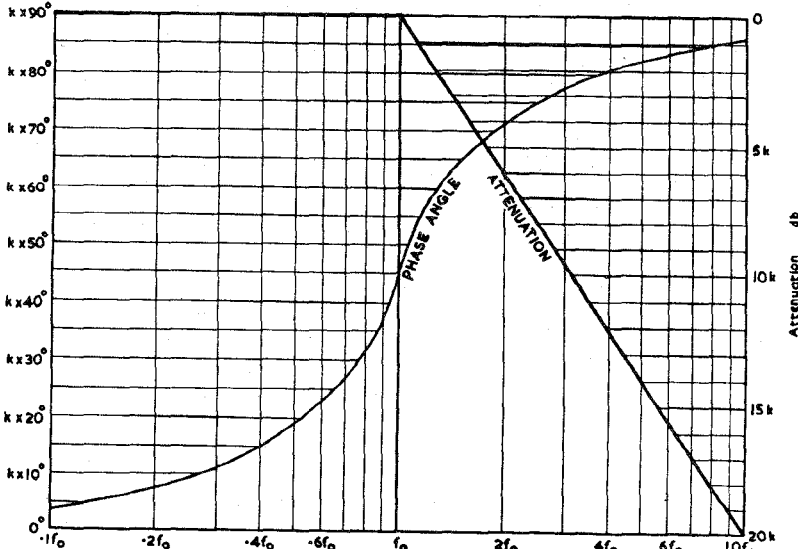

FIG. 7.55 A

Fig. 7.55A. Phase angle for attenuation characteristic as shown; $k = 1$ for 6db/octave, $k = 2$ for 12db/octave etc. (Based on Bode, Refs. H6, H11).

On account of this fairly close agreement between the phase angle characteristics for the two cases, it is possible to replace the actual attenuation characteristic by an approximate " straight-line " equivalent. Where the attenuation characteristic does not have constant slope throughout, it may be replaced by a succession of intersecting straight lines having slopes of zero or any multiple of 6 db/octave, either positive or negative (e.g. Fig. 7.55B). These, in turn, may be resolved into " semi-infinite "* lines of constant slope, as in Fig. 7.55C. The phase characteristics may be derived by the use of Fig. 7.55A for each junction. The resultant phase angle characteristic may be determined by adding these individual characteristics.

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*A " semi-infinite " straight line is one commencing from a definite point and proceeding to infinity in one direction only.

More accurate results may be obtained in some cases by the use of phase angle curves for attenuation characteristics having a sloping portion and a horizontal portion, as given by Bode (Ref. H11, Chapter 15).
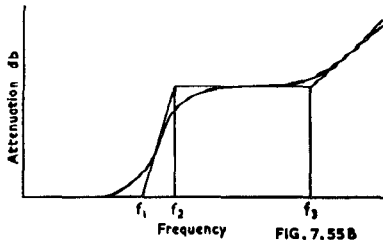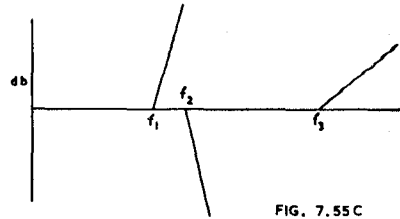

Fig. 7.55B. *Attenuation characteristic with straight line approximation (based on Bode, Ref. H11).*
Fig. 7.55C. *Semi-infinite lines of constant slope corresponding to straight line approximations in Fig. 7.55B (based on Bode, Ref. H11).*

More accurately again, **when the slope of the attenuation characteristic is varying,** the phase angle at any frequency may be determined from a measurement of the amplitude characteristic over a wide frequency range. This procedure is facilitated by the use of special graph paper, plotting the slope of the amplitude characteristic in db/octave against a function of frequency, then measuring the area under the curve and so enabling the slope at one point to be determined (see Ref. A29).

When the slope of the attenuation characteristic reaches a maximum value over a limited frequency range and is less than this value at lower and higher frequencies, the phase angle at the point of maximum slope is less than that indicated by the slope, while at frequencies below and above the region of higher slope the phase angle is greater than that indicated by the slope.

Simple method for determination of phase angle (Ref. H18)

A very simple approximate method, which gives the total phase angle, is based on the straight-line approximate characteristics (Fig. 7.55D). The phase angle displacement at the cut-off frequency f_0 is 45°; the approximate straight-line phase characteristic is taken as a tangent to the actual curve at the point of greatest slope, with sharp bends at the intersections with 0° and 90°. The maximum error is about 10°.
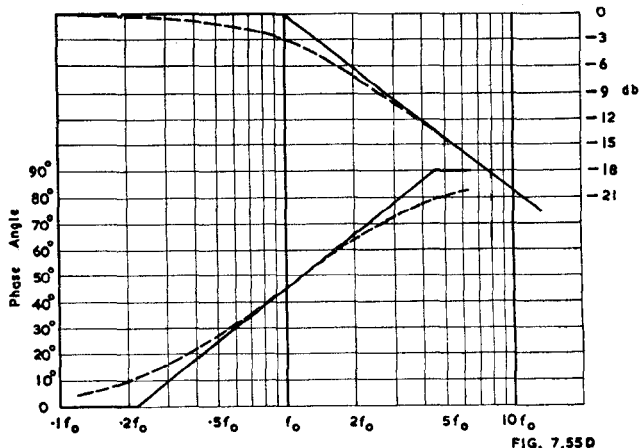

Fig. 7.55D " Straight line " *approximate characteristics with an attenuation slope of 6db/octave (actual characteristics shown with broken lines) and corresponding phase angle characteristics (Ref. H18).*

A typical attenuation characteristic is shown in Fig. 7.55E with three component " semi-infinite " slopes.

The corresponding phase angle characteristics are shown in Fig. 7.55F, where 1 and 2 are to be added, and 3 is to be subtracted to give the resultant. In each case the position of the component phase characteristic is determined by the relationship that the cut-off frequency on the attenuation characteristic gives 45° phase angle.
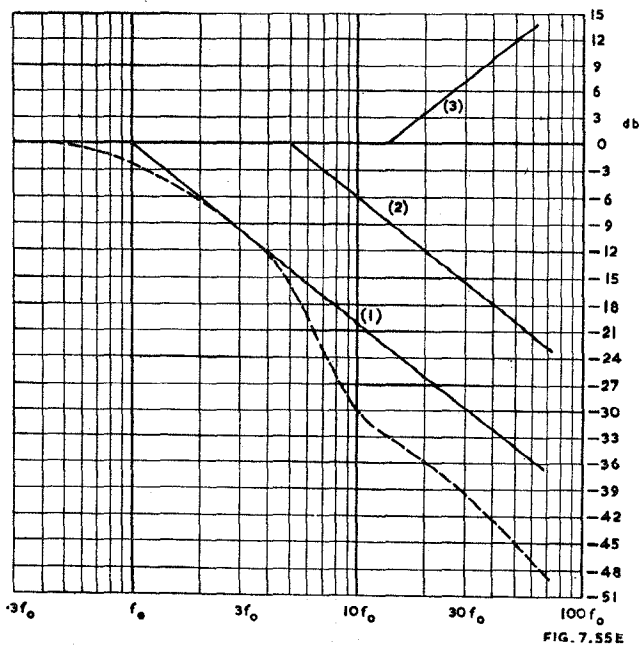

FIG. 7.55E

Fig. 7.55E. *Typical attentuation characteristic of amplifier with feedback (broken curve) and component " semi-infinite " slopes (1, 2 and 3) from which Fig. 7.55A is constructed (Ref. H18).*

The broken line in Fig. 7.55F is the actual phase characteristic, indicating reasonable accuracy for the method. In all cases the actual characteristic tends to " round the corners " of the approximate characteristic.

The same curves (Figs. 7.55D, E and F) may be used for the low frequency attenuation characteristics by inverting the frequency scales.

(iv) Design of 1 and 2 stage amplifiers

A **single stage** resistance capacitance coupled amplifier has $+ 90°$ phase angle displacement at zero frequency due to the grid coupling condenser. The screen and cathode by-pass condensers may cause phase angle displacements up to about $+ 65°$ and $+ 30°$ respectively (see Figs. 12.11B and 12.3C) but these come back to zero again at very low frequencies. Consequently the total phase angle displacement is normally less than 180° at any frequency. At the high frequency end the maximum phase angle displacement is $- 90°$. Such an amplifier is stable under all conditions.

A **two stage** r.c.c. amplifier has twice the phase angle displacement of a single stage amplifier, if both stages are identical. It may therefore produce peaks of frequency response due to positive feedback at low and high frequencies, and care is necessary in design—see Sect. 3(vi) below. It is desirable to " stagger " the low frequency attenuation characteristics by choice of widely different time constants for the grid condensers and following grid resistors ; the larger the ratio, the larger will

be the permissible feedback factor.   One or both cathode by-pass condensers may be omitted, or the by-passed section of R_k may be reduced in resistance.   One or both screen dropping resistors may be reduced in resistance, or replaced by a voltage divider with other consequential adjustments.   The values of the cathode and screen by-pass capacitors may be adjusted so that their frequencies of maximum phase angle are " staggered," preferably in the ratio of at least 20 to 1.

The high frequency peak may be reduced by staggering the high frequency response of the two stages, for example by shunting a capacitor from one plate to earth and increasing the capacitance experimentally until the peak is sufficiently reduced. For an exact design method see Sect. 3(vii) below.

The phase angle displacement is increased by the inclusion of an iron-cored transformer within the feedback loop, also by any resonance effects, and such cases should be treated by the method described for multi-stage amplifiers.
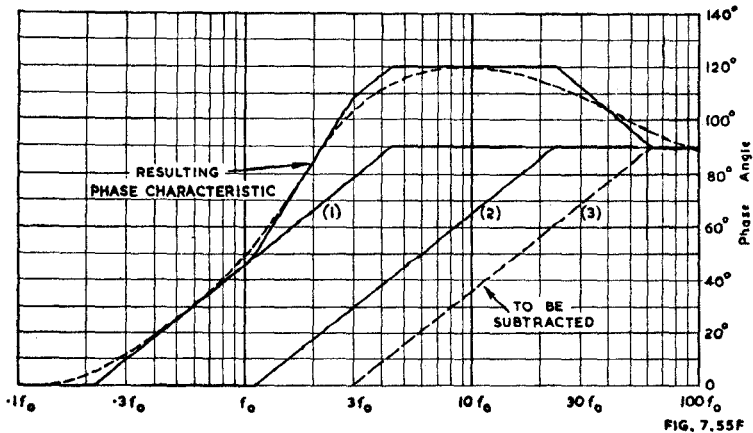

Fig. 7.55F.   *Phase angle characteristic derived from Fig.* 7.55E (*Ref.* H18).

(v) Design of multi-stage amplifiers

In the design of multi-stage amplifiers, Bode's method (A) is most commonly used or its simplification by Learned (B).   This method is intended primarily for use with amplifiers having only one feedback path.

The addition of subsidiary feedback to a single feedback loop leads to Duerdoth's multiple feedback system (E) which is based on a modified stability margin (C) and the theory of the summation of attenuation characteristics and subsidiary voltages (D).   Duerdoth's system is capable of providing a greater feedback factor than Bode's method, with the same minimum value of stability margin in each case.   Some delightfully simple applications of Duerdoth's method are possible and it seems that any amplifier with a single feedback loop can be improved as regards stability by the addition of one or more subsidiary feedback paths.   The latter, in their simplest form, may be merely unbypassed cathode resistors.

With any multi-stage amplifier, an important feature is the **degree of damping on transients**.   Insufficient experimental work has been carried out to indicate what degree of damping is desirable in a-f amplifiers.   A reasonably safe inference is that damping heavier than critical damping is undesirable, because it results in sluggish uptake.   Whether critical damping—see Sect. 3(vii)C—is desirable, or some lighter degree of damping is preferable, is an unsolved problem which can only be tackled on an experimental basis.   It is stated by those who advocate a light degree of damping that this gives very rapid uptake, and the overshoot which occurs is at an ultrasonic frequency which is certainly inaudible, and in any case would be very seriously attenuated by the loudspeaker.   On the other hand, it is well known that some forms of ultrasonic parasitic oscillation give rise to objectionable reproduction.

Design tests

It is highly desirable to measure and plot both the frequency response and phase angle over a sufficiently wide frequency range ; both curves may be plotted on the same sheet of logarithmic graph paper.   The required frequency range is the useful frequency range of the amplifier plus (at each end) one octave for each 10 db of feedback plus at least one octave.

When it is desired to achieve the maximum degree of feedback it is essential to plot the polar (Nyquist) diagram as in Figs. 7.53B and 7.54.   For a method of measuring the phase angle see Refs. H7, H19, H20.

An amplifier may be tested for transient response by applying a rectangular waveform to the input and observing the waveform of the output with a C.R.O. having good frequency and phase characteristics up to the maximum test frequency.   If there is any oscillatory response or " overshoot " the feedback may be reduced or some modification made to the feedback loop to provide the desired degree of damping.

Another useful design test is to overload the amplifier with input voltages of various frequencies, and to determine the level to which the input must be reduced to return to normal linear operation (Ref. A29 discussion).
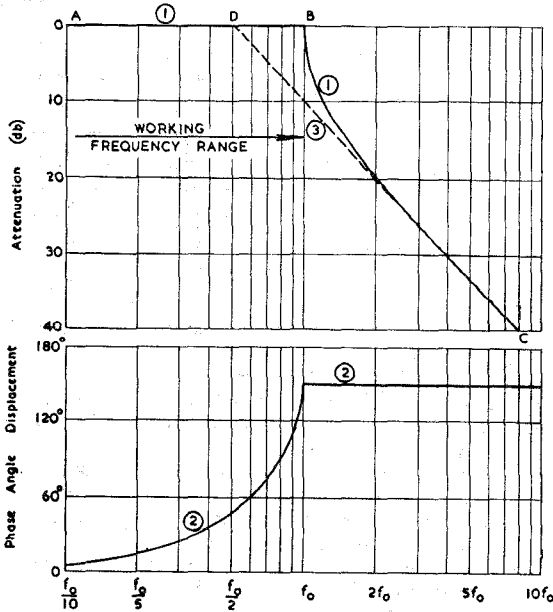

FIG. 7.56 A

Fig. 7.56A.   Bode's method—Curve (1) attentuation characteristic and (2) phase angle displacement characteristic for constant 150° phase angle displacement above upper working frequency f_o ; curve (3) constant slope 10 db/octave (Ref. H11).

Alternative methods of design

(a) One method sometimes used is the " cut and try " method of constructing an amplifier and then applying feedback with the feedback factor increasing in steps until instability occurs, and finally decreasing the feedback to provide a safe margin.

If instability occurs with only a small degree of feedback, there are devices which may be experimented with, such as

1. Shunting the primary of the output transformer by a condenser.
2. Connecting a very small condenser from the plate of one of the earlier stages to earth.

3. Shunting the feedback resistor by a condenser, or the more elaborate network of Fig. 7.59C.

(b) A preferable alternative is to measure and plot the frequency response curve with and without feedback, over a frequency range sufficiently wide to include both low and high frequency peaks. The feedback factor may be increased until the response with feedback is greater than that without feedback by a predetermined amount (say from 1 to 6 db) at any frequency.

(c) Alternatively the amplifier may be designed with " staggered " frequency response. In a 2-stage amplifier, there should be one stage with wide, and the other with a narrow frequency response ; in a 3-stage amplifier there may be two stages with wide and one with narrow frequency response. The use of " maximal flatness " —as described in Sect. 3(vii)A and B—results in oscillatory response to transients, and the feedback factor should be reduced to half the specified value if it is desired to produce critical damping. A preferable arrangement is " compensated " critical damping as described in Sect. 3(vii)C.

(A) **Bode's method** (Refs. H6, H11)

In accordance with Bode's method, the usual procedure is to adopt a constant phase angle displacement, expressed in the form of an attenuation characteristic, as the safety margin. For example, Fig. 7.56A shows the attenuation characteristic BC of Curve 1 for a constant phase angle displacement of 150°, thus giving a constant safety margin of 30°. The ultimate slope of this characteristic is 10 db/octave, which is extended upwards in Curve 3 to point D. The characteristic ABC gives an additional octave of working frequency range as compared with characteristic ADC and is therefore to be preferred if the maximum possible feedback is to be used, together with an adequate safety margin.
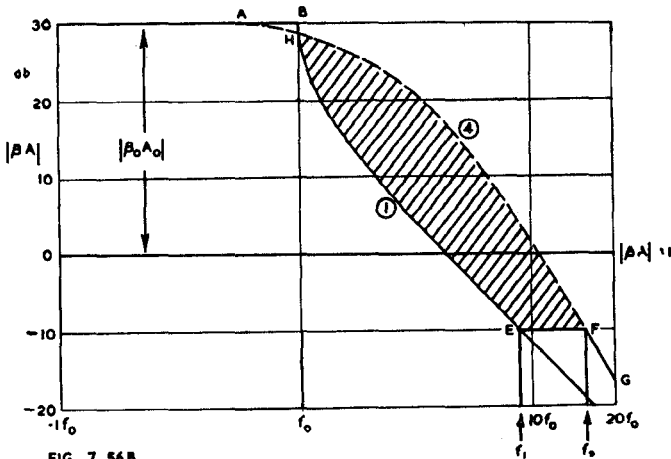

FIG. 7.56B

Fig. 7.56B. Bode's method—Curve (1) as in Fig. 7.56A ; (2) normal attenuation characteristic of typical amplifier. Shaded portion indicates additional attenuation required in the feedback loop.

For convenience in feedback design, Curve 1 has been redrawn in Fig. 7.56B with 0 db corresponding to $|\beta A| = 1$, for 30 db of feedback at the mid-frequency. Curve 4 is the normal attenuation characteristic of a typical amplifier, so that the shaded portion indicates the additional attenuation required in the feedback loop to produce the desired curve 1. The shaded portion is limited at the lower end by the line EF which provides a safety margin (here 10 db) below $|\beta A| = 1$. The desired characteristic is therefore BEFG, where FG is portion of the normal attenuation characteristic of the amplifier. The safety margin is therefore a constant angle of 30° from B

to E, and a constant attenuation of 10 db from E to F   The effect of the horizontal " step " EF is to limit the phase angle displacement in this region, to a value less than 180°, which will hold provided that

$$\frac{f_2}{f_1} = \frac{\text{ultimate slope of actual characteristic (Curve 4) in db/octave}}{\text{ultimate slope of desired characteristic (Curve 1) in db/octave}}$$

Since point F is fixed by the actual attenuation characteristic, the position of point E, and hence Curve 1, will be determined by the ratio f_2/f_1.

Methods for providing the " step " in the characteristic are described in (B) below.

The following relations hold, provided that the ultimate slope of curve 1 is 10 db/octave and the safety margin of the " step " below $|\beta A| = 1$ is 10 db.

Let n = ultimate slope of actual attenuation characteristic, in db/octave

$|\beta_0 A_0|$ = desired value of $|\beta A|$ at the mid-frequency

and f_2 = frequency at which the actual attenuation is equal to $(|\beta_0 A_0| + 10)$ db,

then $f_1 = f_2 . (10/n)$

number of octaves between f_0 and $f_1 = |\beta_0 A_0|/10$

"   "   "   " f_1 and $f_2 = 3.32 \log(n/10)$.

The frequency ratio may be derived from the number of octaves by the relation—

frequency ratio = antilog (0.301 × number of octaves)

or by the use of the table below :

Frequency Ratio, Octaves and Decades

Frequency Ratio	No. of Octaves	No. of Decades	Frequency Ratio	No. of Octaves	No. of Decades
1.07	0.1	0.03	5.28	2.4	0.72
1.15	0.2	0.06	6.07	2.6	0.78
1.23	0.3	0.09	6.97	2.8	0.84
1.32	0.4	0.12	8.00	3.0	0.90
1.42	0.5	0.15	9.18	3.2	0.96
1.52	0.6	0.18	10.0	3.32	1.00
1.63	0.7	0.21	10.6	3.4	1.02
1.74	0.8	0.24	12.1	3.6	1.08
1.87	0.9	0.27	13.9	3.8	1.14
2.00	1.0	0.30	16.0	4.0	1.20
2.30	1.2	0.36	18.4	4.2	1.26
2.64	1.4	0.42	21.2	4.4	1.33
3.03	1.6	0.48	24.3	4.6	1.39
3.48	1.8	0.54	27.9	4.8	1.45
4.00	2.0	0.60	32.0	5.0	1.51
4.60	2.2	0.66			

Frequency ratio   = antilog (number of decades)

= antilog (0.301 × number of octaves)

Number of decades = log (frequency ratio)

= 0.301 × number of octaves

Number of octaves = 3.32 × log (frequency ratio)

= 3.32 × number of decades.

Relationship between db/octave and db/decade

db/octave	3	6	9	10	12	15	18
db/decade	10	20	30	33.3	40	50	60

In addition to the attenuation of the shaded area in Fig. 7.56B, it is also necessary to boost the gain near the limits of the working frequency band to provide nearly constant gain (area AHB) and thereby achieve almost zero phase angle at the cut-off frequency f_0. This may be accomplished by the circuit of Fig. 7.56C in which the Q of the $L_1 C_1$ circuit should be unity, as given by

$$L_1 = 1/\omega^2 C_1 \text{ and } r = 1/\omega_0 C_1$$

where r = series resistance of L_1. More complicated but more satisfactory networks (low-pass or band-pass " Wheeler " networks) are described in Ref. A29. These

have an ultimate attenuation of 6 db/octave and a phase angle displacement of approximately 90° at all frequencies outside the working band. For other methods see Ref. H11.

The low frequency end may be treated in a similar manner throughout.

Means for producing the desired attenuation characteristics are described by References H6, H10, H11. See also (B) below.

This method of design requires the amplifier to have a very much wider frequency range before feedback is applied than it would need without feedback. The extension is approximately one octave in each direction for each 10 db reduction of gain by feedback plus a sufficient margin of safety (say 1 octave at each end). Thus an amplifier to cover from 40 to 15 000 c/s with 30 db reduction of gain by feedback would require to have a frequency response without feedback (at 40 db down) from 2.5 to 240 000 c/s. This actually applies to the transmission characteristics of the whole feedback loop (amplifier and feedback network together).

FIG. 7.56C

Fig. 7.56C. Simple network in plate circuit to give more nearly constant gain over the working frequency range (based on Ref. A29).

(B) Simplified treatment by Learned (Ref. H9)

The following treatment is based on the simplified assumption that the attenuation characteristics are straight lines. This is approximately correct except in the octave immediately above and below the useful frequency band of the amplifier, where the method gives a wider safety margin than is necessary. The method does not, therefore, give the same degree of feedback obtainable with the precise method of Ref. H6. At the same time it is quite suitable for general use.
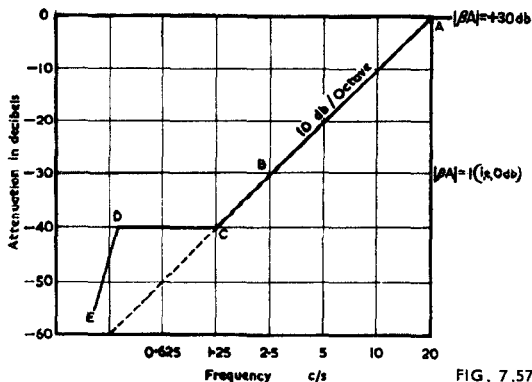

Fig. 7.57. Low frequency attenuation characteristics of multi-stage amplifier with feedback (simplified treatment).

Low frequency attenuation characteristics

As an example for illustrating the general principle, take an amplifier with nominally flat frequency response down to 20 c/s (actually it will be -2 or -3 db at this frequency). Let the loop amplification $|\beta A|$ be 30 db at useful frequencies.

Draw the attenuation characteristic with a slope of 10 db/octave from 20 c/s downwards (line ABC in Fig. 7.57). Point B is where the value of the loop amplification becomes unity. Extend the line AB to C which is at a level 10 db below $|\beta A| = 1$. At C, insert a horizontal " step " CD, the length of which will be given later. Beyond D the attenuation will fall at a rate of 12, 18, 24, 30 db or more depending on the circuit ; each single time constant or non-resonant transformer contributes 6 db, while each resonant circuit contributes 12 db. The frequency ratio between points
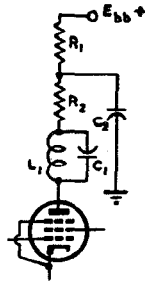
C and D is made equal to the ratio of the slopes of *DE* and *AC*, which is equal to the slope of *DE* divided by 10. The line *ABCDE*, thus determined, is to be used as a guide to the low frequency attenuation characteristic of the amplifier. In reality, of course, the sharp corners will be rounded—the important features are that the slope of the *AC* region should not exceed 10 db/octave, and that the step should be sufficiently long.
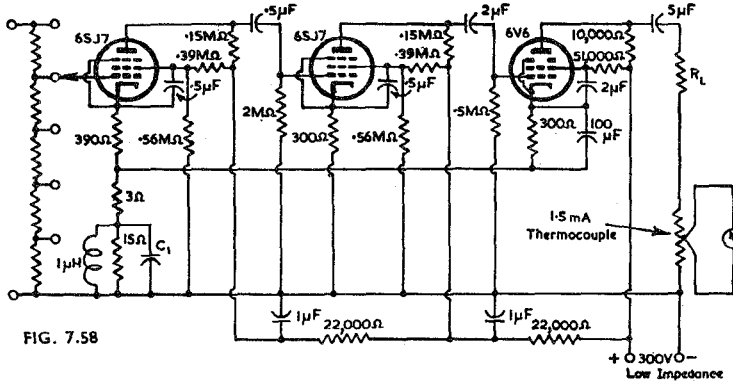

FIG. 7.58

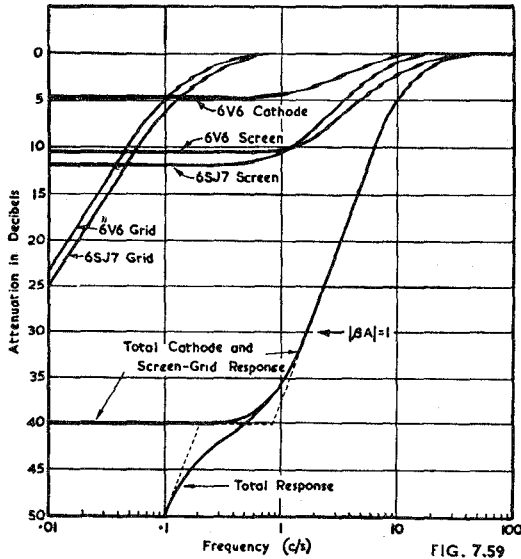Fig. 7.58. *Three stage amplifier with negative feedback illustrating design method (Ref. H9).*

Fig. 7.59. *Low frequency attenuation characteristics of Fig.* 7.58 *(Ref. H9).*

Example

An example of an amplifier designed in accordance with this method is Fig. 7.58 (Ref. H9). The low frequency attenuation characteristics are given in Fig. 7.59. The 6V6 cathode, 6V6 screen and 6SJ7 screen characteristics are designed to provide a very close approach to a total slope of 10 db/octave with a limiting attenuation of 40 db, so providing the basis of a step. The two grid condensers are designed to give effective attenuation beyond the limit of the step. It is evident that it is possible to design an amplifier which provides the necessary low frequency attenuation charac-

teristics, including the step, without using any resonant circuits. The tolerances on the frequency-dependent components (R and C) must be small.

The feedback loop does not include the plate circuit of the third stage, so that the slope of the high-frequency attenuation characteristic is nominally 12 db/octave. The resonant circuit (incorporating C_1) in the feedback path is employed to stabilize the frequencies in this region by providing a step in the high-frequency attenuation characteristic.

Corrective networks

The design of simple corrective networks is well summarized in Ref. H9. More complicated designs incorporate LCR 2-terminal networks as the plate load impedances and (some) cathode bias impedances (Ref. H11). See also Ref. J16.

One of the simplest and most effective methods of providing the required step in the high frequency attenuation characteristic is the RC network shunting the plate load resistor of the pentode V_1 in Fig. 7.59A (based on Ref. H9). The ultimate attenuation of the step in decibels is given approximately by

$$\text{attenuation} \approx 20 \log (R_1 + R)/R.$$

The cut-off frequency is given by

$$f_1 = 1/2\pi R_1 C$$

and the " flattening-out " frequency by

$$f_2 \approx 1/2\pi RC$$

where $R_1 = R_L R_g/(R_L + R_g)$
and the " flattening-out " frequency is defined as the frequency at which the attenuation is 3 db less than the ultimate attenuation of the step.
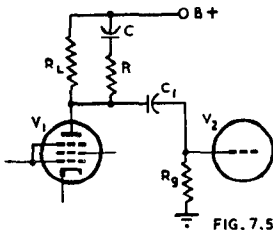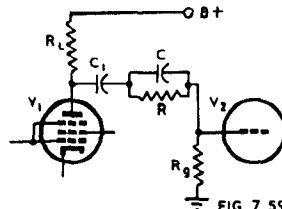

FIG. 7.59A   FIG. 7.59B

Fig. 7.59A.   *Circuit to provide step in high frequency attenuation characteristic (based on Ref. H9).*

Fig. 7.59B.   *Circuit to provide step in low frequency attenuation characteristic (based on Ref. H9).*

The equivalent to Fig. 7.59A for the low frequency end is provided by Fig. 7.59B :
Step attenuation $\approx 20 \log (R + R_g)/R_g$
Cut-off frequency $f_1' = 1/2\pi C(R_L + R_g)$
Flattening-out frequency $f_2' \approx 1/2\pi RC$
The value of C_1 is selected to give negligible attenuation at f_2' ; say

$$C_1 \approx 5/[\pi f_2'(R + R_L + R_g)].$$

With these step circuits it is desirable to have as many separate circuits as possible, each with a small step.

Fig. 7.59C.   *Feedback path including CR network to give a step in the attenuation characteristic, and thereby to improve stability (Ref. H18).*
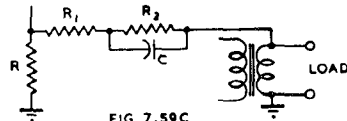

FIG. 7.59C

A step in the attenuation characteristic may also be provided by an LCR circuit in the feedback path, or more simply by a CR circuit as in Fig. 7.59C. Here R may be the cathode resistor of the first valve, $R_1 + R_2$ take the place of the usual feedback resistor, and C effectively increases the value of β at high frequencies. This is a most useful device to improve the stability of a feedback amplifier. A simpler form, omitting R_1, is sometimes sufficient ; this is similar to Flood's " compensated feedback " Sect. 3(vii)C.

(C) **Duerdoth's stability margin** (Ref. A29)

Duerdoth adopts a pre-determined value (e.g. 6 db) of the rise of gain with feed-back at high and low frequencies as a stability margin over values of βA from 0.3 to some value less than 2.0. At higher values of βA he adopts an angular stability margin (e.g. 15°) while below $\beta A = 0.3$ there is no danger of instability. An amplifier designed in accordance with his method might have a high-frequency characteristic such as OABCDEFG in Fig. 7.54. See discussion on stability and tolerances in components with this characteristic in Sect. 3(ii) above. One method for achieving such a shape of characteristic is given in Sect. 3(v)E below.

(D) **Summation of attenuation characteristics and subsidiary voltages**

When two voltages have to be added, as for example with a loop feedback voltage and a subsidiary voltage connected in series, the attenuation characteristic can only be derived by adding the voltages at selected frequencies, with due regard to the phase angle between them.

The phase angle between the two voltages is a function of the difference in slope when plotted as attenuation characteristics, provided that the slopes are constant :

Slope difference	2	4	6	8	10	12	14	16	18 db/octave
Phase angle diff.	30	60	90	120	150	180	210	240	270 degrees

For example : If one characteristic has a slope of 4 db/octave and the second 12 db/octave, the slope difference is 8 db/octave, and the phase angle between the voltages is therefore 120°.

Take the intersection of the two characteristics as the origin (as in Fig. 7.60) and assume that 0 db = (say) 1 volt—both assumptions being for convenience.

Let x be the voltage contributed by the lower slope characteristic at a certain frequency. It may readily be shown that the voltage contributed by the high slope characteristic at this frequency is x^a, where a is the ratio of slopes in db/octave. These two voltages, x and x^a must then be added vectorially, the angle between them being as given by the table above.

The magnitude of the combined characteristic may then be plotted as in Fig. 7.60, and the values of the phase angle may be marked along it.

The method of deriving the magnitude and phase angle of the resultant is illustrated by Fig. 7.61 for the condition where the higher slope characteristic has a slope of 12 db/octave, corresponding to 180°. Vector OA, having magnitude x and phase angle
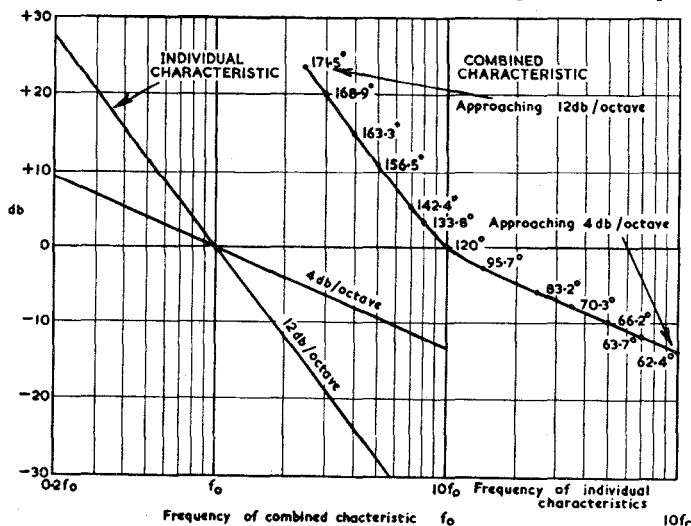

FIG. 7.60

Fig. 7.60. *Illustrating summation of attenuation characteristics (based on Ref. A29).*

ϕ indicates the lower slope characteristic, the value of ϕ in degrees being given by 15 × slope in db/octave. Vector OB, having magnitude x^a and phase angle $180°$, indicates the higher slope characteristic with a slope of 12 db/octave. For example, when the slopes are 4 and 12 db/octave, $a = 3$ and $\phi = 60°$.

The magnitude of the resultant is given by

$$\text{Resultant} = \sqrt{(x \sin \phi)^2 + (x^a - x \cos \phi)^2}.$$

The phase angle θ of the resultant is given by

$$\theta = \tan^{-1} \frac{x \sin \phi}{x \cos \phi - x^a} .$$

For ease in calculations the following table has been derived for use in all normal cases when the phase angle between the two voltages is a multiple of 30° and less than 180° and when the higher slope is 12 db/octave.

Difference in phase angle*	Resultant	Phase Angle θ of Resultant
30°	$x\sqrt{1 + 1.73x^{0.2} + x^{0.4}}$	$\tan^{-1} -- \dfrac{1}{2x^{0.2} + 1.73}$
60°	$x\sqrt{1 + x^{0.5} + x}$	$\tan^{-1} -- \dfrac{1.73}{2x^{0.5} + 1}$
90°	$x\sqrt{1 + x^2}$	$\tan^{-1} -- 1/x$
120°	$x\sqrt{1 - x^2 + x^4}$	$\tan^{-1} \dfrac{1.73}{1 - 2x^2}$
150°	$x\sqrt{1 - 1.73x^5 + x^{10}}$	$\tan^{-1} \dfrac{1}{1.73 - 2x^5}$

*between the two voltages, i.e. $180° - \phi$.

A similar procedure can, of course, be applied to any other value of maximum slope.

Procedure when lower slope is zero

The foregoing procedure cannot be used when one of the two attenuation characteristics has a slope of zero, that is when one is a fixed voltage. In this case the procedure is as follows.
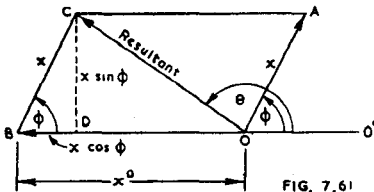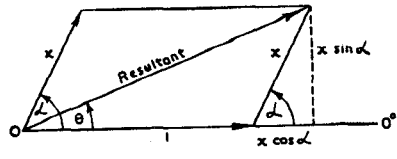

FIG. 7.61   FIG. 7.62

Fig. 7.61. Vector relationships in summation of attenuation characteristics.
Fig. 7.62. Vector relationships when one characteristic has zero slope.

As previously, the origin is the point of intersection of the two characteristics, and its level is taken as 0 db equal to 1 volt. The vector relationships are shown in Fig. 7.62.

$$\text{Resultant} = \sqrt{x^2 + 1 + 2x \cos \alpha}$$

$$\text{Phase angle of resultant} = \theta = \tan^{-1} \frac{x \sin \alpha}{1 + x \cos \alpha}$$

where $x =$ voltage at a selected frequency, on the sloping characteristic
and $\alpha =$ phase angle of sloping characteristic (i.e. angle in degrees $= 15 \times$ slope in db/octave).

Angle α	Resultant	Phase angle θ of Resultant
30°	$\sqrt{x^2 + 1.73x + 1}$	$\tan^{-1} \dfrac{x}{2 + 1.73x}$

Angle α	Resultant	Phase angle θ of Resultant
60°	$\sqrt{x^2 + x + 1}$	$\tan^{-1} \dfrac{1.73x}{2 + x}$
90°	$\sqrt{x^2 + 1}$	$\tan^{-1} x$
120°	$\sqrt{x^2 - x + 1}$	$\tan^{-1} \dfrac{1.73x}{2 - x}$
150°	$\sqrt{x^2 - 1.73x + 1}$	$\tan^{-1} \dfrac{x}{2 - 1.73x}$
165°	$\sqrt{x^2 - 1.93x + 1}$	$\tan^{-1} \dfrac{0.26x}{1 - 0.97x}$

An example of the summation of two attenuation characteristics, one having zero slope and the other having a slope of 10 db/octave, is given in Fig. 7.63. The resultant is asymptotic to 10 db/octave and to the horizontal characteristic, but reaches a minimum value at the frequency of the point of intersection O. The phase angle is marked at points along the resultant, and at all frequencies the phase angle is less than that of the 10 db/octave characteristic, the effect becoming more pronounced
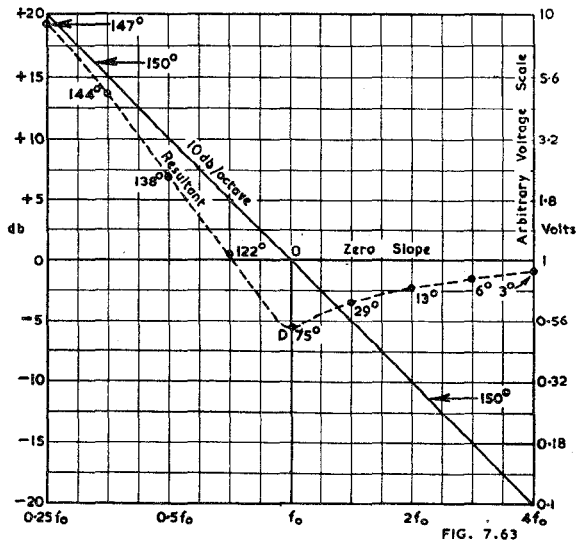

Fig. 7.63. *Summation of two attenuation characteristics having slopes of zero and 10 db/octave.*

at higher frequencies, particularly beyond point D. The fact that the slope of the resultant in Fig. 7.63 exceeds 12 db/octave to the left of point D, has no effect whatever on the stability as indicated by the angles. It is therefore obvious that the relationship between phase angle and slope of the attenuation characteristic which applies to minimum phase shift networks, does not apply here.

A similar calculation has been made for a slope of 11 db/octave, Fig. 7.64.

The position of point D on the resultant characteristic is a function of the slope of the attenuation characteristic.

slope in db/octave	2	4	6	8	10	11
height of point D	+ 5.7	+ 4.7	+ 3	0	− 5.6	− 11.6 db.

(E) Duerdoth's method employing multiple feedback paths (Ref. A29)

In accordance with this method, one or more additional subsidiary feedback voltages are introduced into the feedback loop so as to decrease the phase angle over the
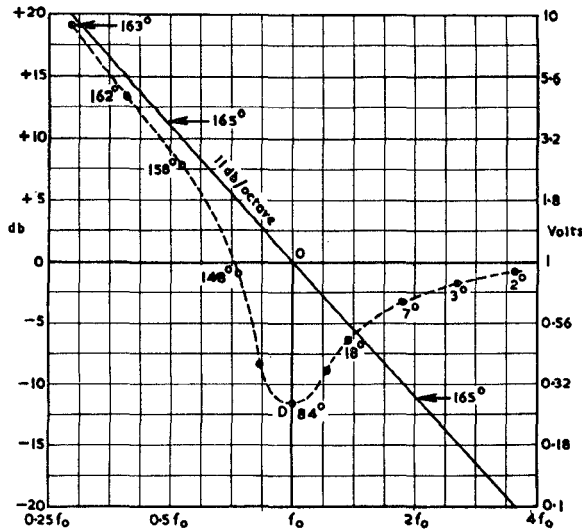
Fig. 7.64. Summation of two attenuation characteristics having slopes of zero and 11 db/octave.

attenuation portion of the amplitude characteristic without appreciably affecting the characteristics within the working frequency range.

With subsidiary feedback applied over the first stage of a 2-stage amplifier and loop feedback over the whole amplifier, the ratio of gain with and without feedback is given by

$$\frac{A'}{A} = \frac{1}{1 - \beta_1 A_1 - \beta_2 A} \tag{1}$$

where A = overall amplification without feedback
$\quad\quad A'$ = overall amplification with feedback
$\quad\quad A_1$ = amplification of first stage without feedback
$\quad\quad \beta_1$ = value of β for subsidiary feedback over first stage only
and $\quad \beta_2$ = value of β for loop feedback over whole amplifier.

It is evident that when subsidiary feedback is used, the several feedback voltages must be added at their common point. This holds even when the subsidiary voltage is entirely inside the feedback loop.

Under the same conditions, the ratio of distortion in the final stage with and without feedback is given by

$$\frac{D'}{D} = \frac{1 - \beta_1 A_1}{1 - \beta_1 A_1 - \beta_2 A} = \frac{A'}{A}(1 - \beta_1 A_1) \tag{2}$$

Thus the application of negative subsidiary feedback to the first stage increases the distortion in the final stage in the same proportion that the gain of the first stage is reduced, provided that the overall feedback loop is unchanged.

If, however, the subsidiary feedback is positive, the gain of the first stage will be increased and the distortion in the final stage will be reduced further. The use of positive subsidiary feedback need not present any stability problems, since the stability criterion is dependent on $(\beta_1 A_1 + \beta_2 A)$. See Sect. 2(xi) and Refs. F11, F12.

With multiple loop amplifiers, the usual Nyquist stability criterion applies to any loop, provided that the amplifier remains stable when the particular loop is broken. Even with amplifiers which are always stable, the Nyquist diagrams obtained by measurements of the several loops in turn may have different shapes and the definition of stability margin becomes meaningless. However, if a mesh of the amplifier can be found which, if broken, simultaneously breaks all the loops, then there is no possi-

bility of oscillation due to the disconnection and the definition of stability margin remains applicable (Ref. A29).

When employing multiple feedback paths with summation of attenuation characteristics and subsidiary voltages, the crossing point of the characteristics should be controlled with considerable accuracy. The crossing point will be modified when the gains of the various stages change owing to changes in the valve characteristics with age, or to overloading caused by an excessive input voltage. The latter may be avoided or reduced in more elaborate amplifiers by the addition of a cathode follower stage, operating as a limiter, as part of the first stage. Precautions to be taken in design are described in Ref. A29.

Some applications employing multiple feedback paths are described below. See also Sect. 2(vi)A.
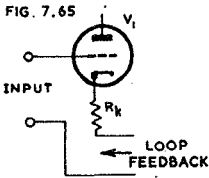
FIG. 7.65

Fig. 7.65. Part of first stage of feedback amplifier showing one method of applying subsidiary feedback (Ref. A29).

(1) One simple application is the circuit of Fig. 7.65 where the signal voltage across the unbypassed cathode resistor R_k is added to the loop feedback voltage. This is an application of the principle of summation of attenuation characteristics as shown in Figs. 7.63 and 7.64. The voltage across R_k is practically constant over the critical region of the high frequency attenuation characteristic, so that its characteristic has zero slope. This method may only be used when the phase angle of the βA locus is always less than 180°, i.e. the slope of the βA attenuation characteristic is less than 12 db/octave. This method does not reduce the distortion in the final stage to the same level as without subsidiary feedback, or to that using one of the alternative methods below. For this reason the subsidiary voltage should be considerably smaller than the loop feedback voltage. However, this method will improve the stability margin of an amplifier and reduce the amplitude of the high frequency peak.
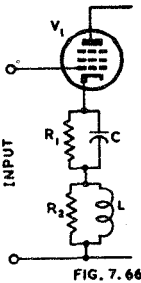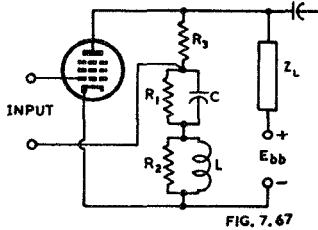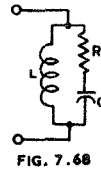

Fig. 7.66. One form of subsidiary feedback (after Ref. A7).
Fig. 7.67. Another form of subsidiary feedback (after Ref. A7).
Fig. 7.68. Two terminal network as cathode load impedance.

(2) One modification of this method, which has only small degeneration in the working range, is shown in Fig. 7.66. This appears to have been first described by Farren (Ref. A7) and applied to one of the stages inside the feedback loop. The phase shift reaches maxima at low and high frequencies outside the working range, beyond which it approaches zero in both directions. The degeneration increases gradually in both directions to the limiting design values provided by R_1 and R_2. This arrangement does not modify the fundamental phase shift between the input and output voltages of the stage due to the phase angle of the plate load impedance.

(3) A further modification by Farren (Ref. A7) is shown in Fig. 7.67 which has the advantage at very high and very low frequencies that it reduces the phase shift between the input and output voltages of the stage due to the phase angle of the plate load impedance which would be characteristic of the stage if there were no subsidiary feedback.
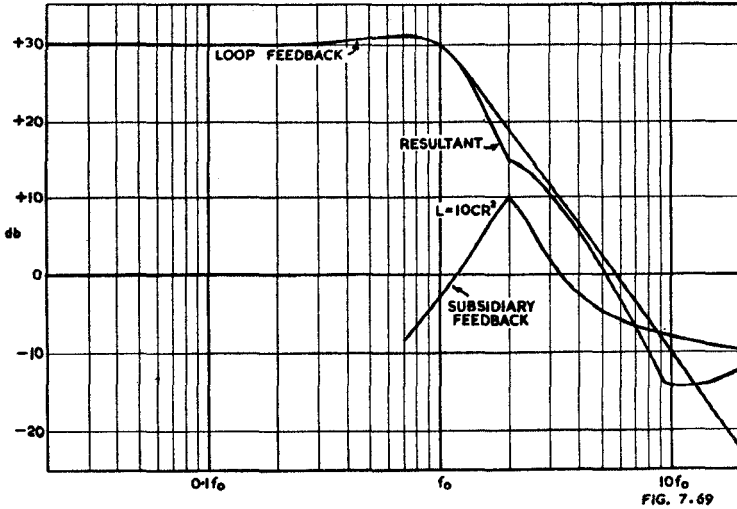

Fig. 7.69. *Summation of loop and subsidiary feedback voltages to provide improved stability (Ref. A29).*

(4) Two terminal networks of the form of Fig. 7.68 are described by Duerdoth as cathode impedances for improving the stability of feedback amplifiers with an ultimate attenuation of 12 db/octave. A value of $L = 10CR^2$ may be used as a first trial and maximum feedback and zero phase angle will then occur at a frequency of approximately

$$f = 0.048/CR$$

where C and R are in farads and ohms respectively. An example of the application of subsidiary feedback on an amplifier with an ultimate attenuation slope of 12 db/octave is given in Fig. 7.69. In this example the peak of subsidiary feedback occurs at twice the maximum frequency of the working band. The stability margin in the critical region has been increased from $10°$ to $36°$ with subsidiary feedback, as shown by the Nyquist diagram (Ref. A29).

It often happens that the addition of the subsidiary voltage results in an increase in phase angle just above the working frequency band where the magnitude of the
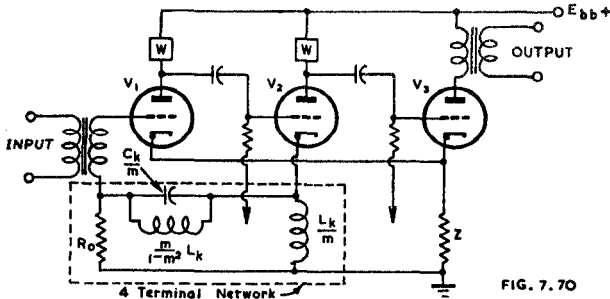

Fig. 7.70. *Three stage amplifier incorporating subsidiary feedback in the form of a 4 terminal network. $m = 0.87$. $W = $ " Wheeler " network (based on Ref. A29).*

subsidiary voltage is increasing with frequency, although a decrease in phase angle occurs where the magnitude of the subsidiary voltage is steady or decreasing with frequency. This increase in phase angle will reduce the angular margin over a range of frequencies and, in the extreme case when the frequency of the peak subsidiary voltage is greater than 3 or 4 times the maximum frequency of the working band, may approach " conditional stability."

A similar technique may also be applied to low frequency attenuation characteristics.

(5) Four terminal networks of the form included in Fig. 7.70 are described by Duerdoth for providing subsidiary feedback in amplifiers with an ultimate attenuation slightly over 12 db/octave. Fig. 7.70 has loop feedback from the cathode of V_3 to the cathode of V_1, main subsidiary feedback from the cathode of V_2 to the grid circuit of V_1, and minor subsidiary feedback due to the unbypassed cathodes of V_1 and V_3. The four terminal network is a high-pass m-derived half-section filter terminated in R_o. The cut-off frequency of the filter is made about twice the maximum frequency of the working band (Ref. A29).
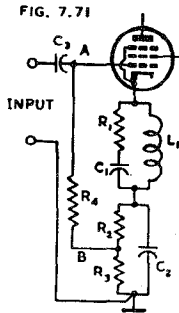
FIG. 7.71

Fig. 7.71. Combination of high and low frequency subsidiary feedback.

(6) A combination of both high and low frequency subsidiary feedback is shown in Fig. 7.71 where $R_1 C_1 L_1$ is the high-frequency network (as in Fig. 7.68) and $R_2 R_3 C_2$ is the low frequency network. The latter provides a subsidiary negative feedback at low frequencies only ; its value will be affected by the source impedance of the input voltage. If transformer coupling is used, the secondary should be connected to points A and B.

(7) In a three stage amplifier with an ultimate slope of about 18 db/octave, the second stage being a phase splitter which introduces little phase shift, some reduction in the total loop phase angle may be achieved by means of a phase shifting network in the feedback line. A subsidiary feedback path is added from the plate circuit of the phase splitter through a high-pass filter to the cathode of the first stage. This voltage is suppressed in the working band by the filter, but above the working band the loss of the network falls and the subsidiary voltage becomes the predominant factor, thus reducing the ultimate slope to 6 db/octave. Complete stability is thus achieved (Ref. A29 Fig. 26).

(8) A third feedback voltage may be added, if desired, to give two subsidiary feedback paths.

(vi) Effect of feedback on frequency response

In the case of a single stage r.c.c. amplifier (assuming perfect cathode and screen by-passing) the feedback merely widens the frequency range without changing the shape of the voltage gain characteristics (Fig. 7.72).

With two such identical stages, the frequency characteristics are as Fig. 7.73. The peaks, which occur as the value of βA is increased, are due to the reduction in effective negative feedback and the development of positive feedback through phase angle shift, which approaches 180° at very low and very high frequencies. F_1 and F_2 are the frequencies giving 0.707 relative voltage gain for each single stage without feedback. Ref. H4.

These peaks may be reduced or eliminated entirely by designing the amplifier with one stage having a much wider frequency range than the other. Design methods to produce " maximal flatness " are described in Sect. 3(vii) and these may be modified to produce critical damping, if desired.

Alternatively a multiple feedback system may be adopted—for example one overall feedback loop in conjunction with an unbypassed cathode resistor in the first stage (Duerdoth's method) as described in Sect. 3(v)E.

See also Sect. 3(iv) and Sect. 2(vi) for the design of two-stage amplifiers.

A special application to a particular two-stage amplifier is Ref. H5.
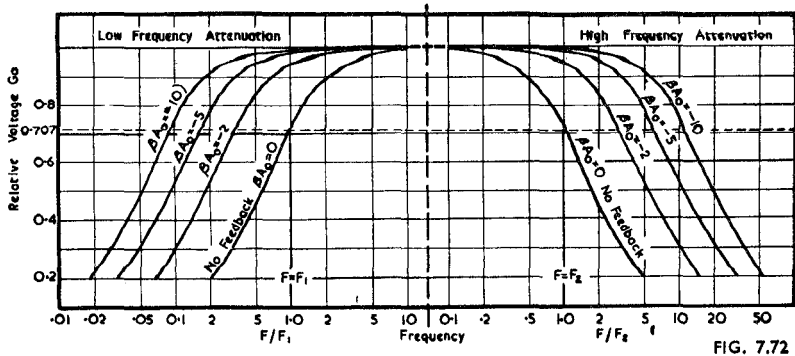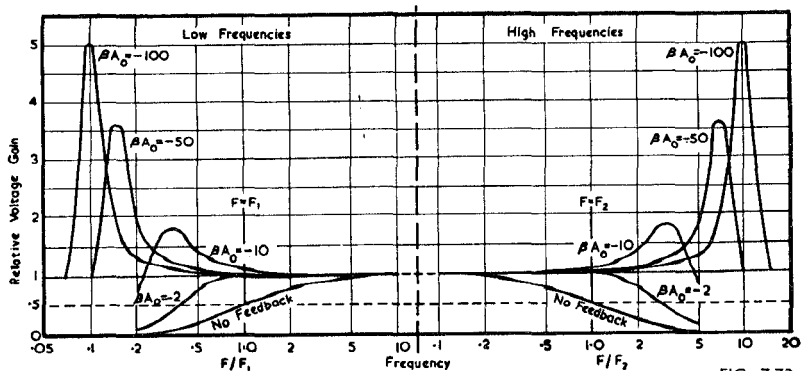
See also Refs. J8, J11.

FIG. 7.72

FIG. 7.73

Fig. 7.72. Relative voltage gain of a single stage r.c.c. amplifier without feedback and with three selected values of feedback (Ref. H4).

Fig. 7.73. Relative voltage gain of two identical r.c.c. amplifier stages, without feedback and with four selected values of feedback (Ref. H4).

Graphical method for two-stage r.c.c. amplifier

An ingenious graphical method has been developed by Barter (Ref. H17), and may be used for determining the R and C values to give any desired height of peaks at low and high frequencies.

(vii) Design of amplifiers with flat frequency response

(A) **Method of H. Mayr** (Ref. H13). See also J13.

A resistance-capacitance-coupled amplifier Fig. 7.74 may be regarded, so far as frequency response is concerned, as a tuned circuit with a Q less than 0.5. The value of this Q is given by

$$Q = \frac{1}{b(d + k) + (1/b)(p + k)} \tag{3}$$

where $b = \sqrt{R_g/R_L}$; $d = C_g/C$; $k = C_c/C$;
$p = C_p/C$ and $C = \sqrt{C_g C_c + C_g C_p + C_c C_p}$.

This method may be applied to multi-stage amplifiers with feedback, it being assumed that all stages have the same resonant frequency $(\omega_0/2\pi)$. The conditions for maximum flat response are then given by :

1 stage $|(A_0/A)| = \sqrt{1 + x^2}$ (4)
$\phi = -\tan^{-1}x$ (5)

$$\text{where } x = \frac{Q}{n}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{6}$$

N.B. n must be greater than 2.

2 stages $|(A_0/A)| = \sqrt{1 + x^4} \tag{7}$

$$\phi = -\tan^{-1}\sqrt{2}x/(1 - x^2) \tag{8}$$

$$\text{where } x = Q_2\sqrt{\frac{Q_1}{nQ_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{9}$$
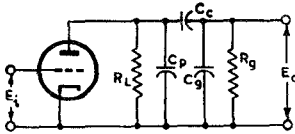
and $\quad Q_1/Q_2 = (n - 1) + \sqrt{n(n - 2)} \tag{10}$

Fig. 7.74. *Resistance-capacitance coupled stage.*

FIG. 7.74

3 stages $|(A_0/A)| = \sqrt{1 + a_2x^4 + x^6} \tag{11}$

$$\phi = -\tan^{-1}(b_1x - x^3)/(1 - b_2x^2) \tag{12}$$

$$\text{where } x = Q_2\sqrt[3]{\frac{Q_1}{nQ_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{13}$$

$$a_2 = \frac{1}{\sqrt[3]{n^2}}\left[\sqrt[3]{\frac{Q_2^4}{Q_1^4}} + 2\sqrt[3]{\frac{Q_1^2}{Q_2^2}}\right] \tag{14}$$

$$b_1 = \frac{1}{\sqrt[3]{n^2}}\left[\sqrt[3]{\frac{Q_1^2}{Q_2^2}} + 2\sqrt[3]{\frac{Q_2}{Q_1}}\right] \tag{15}$$

$$b_2 = \frac{1}{\sqrt[3]{n}}\left[\sqrt[3]{\frac{Q_2^2}{Q_1^2}} + 2\sqrt[3]{\frac{Q_1}{Q_2}}\right] \tag{16}$$

$$Q_1/Q_2 = 2(n - 1) + \sqrt{2n(2n - 3)} \tag{17}$$

and $\quad Q_2 = Q_3. \tag{18}$

N.B. n must be greater than 1.5.

This has also been extended to four stages (Ref. H13).

Symbols used :

A_0 = amplification with feedback at mid-frequency ($\omega_0/2\pi$)

A = amplification with feedback at any frequency ($\omega/2\pi$)

α_0 = amplification without feedback at mid-frequency ($\omega_0/2\pi$)

ϕ = phase angle

n = $1 - \alpha_0\beta_0$ = feedback factor

β_0 = transfer coefficient (negative for negative feedback)

ω_0 = $1/(C\sqrt{R_LR_g})$.

These equations have been plotted in Fig. 7.75 from which it will be seen that the feedback factor has no effect on the shape of the curves for 1 or 2 stages, and only a very slight effect for 3 or 4 stages (curves for limiting values of n are given). Although the curves are shown only for the high frequency limit of the flat top, they may be applied to the low frequency side by changing the sign of the abscissae and, in the case of phase angle, also changing the sign of the ordinates.

This method of design provides the maximum flat frequency response, but the maximum degree of feedback only occurs at the mid-frequency and it falls off rapidly outside the frequency band of the " selective " stage. The feedback becomes zero near the knees of the flat top, and positive at still higher and lower frequencies.

This design ensures stability at all frequencies from zero to infinity.

This method also makes it possible to design for any desired response curve, within the limits of the amplifier. The procedure is to commence with the desired response curve and to determine the response (either modulus or phase angle) corresponding
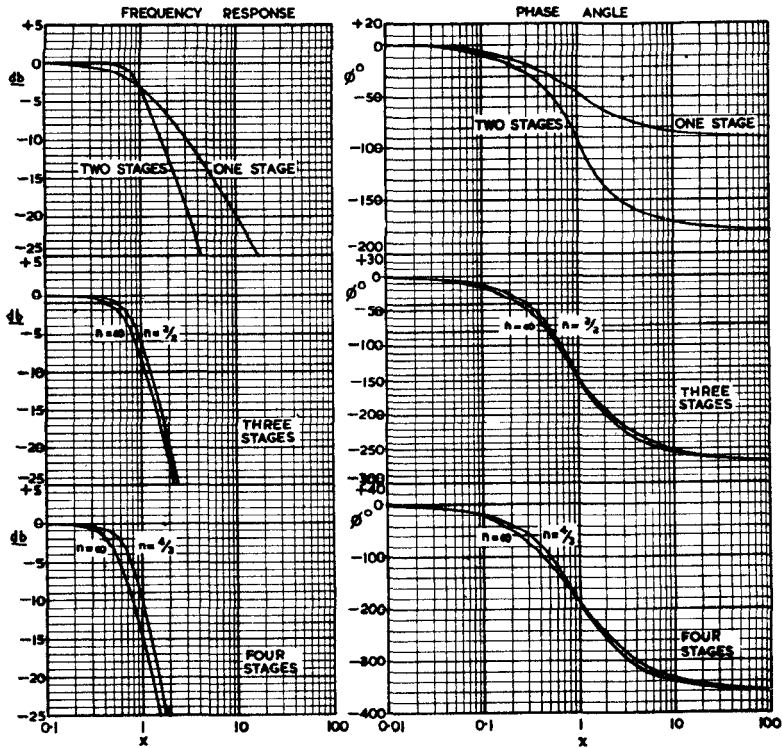
FIG. 7.75

Fig. 7.75. Frequency response and phase angle of frequency response for 1, 2, 3, and 4 r.c.c. stages designed for maximum flat response by Mayr's method (Ref. H13).

to two arbitrary frequencies $(\omega_1/2\pi)$ and $(\omega_2/2\pi)$, one near the lower and the other near the upper frequency limit. Then we read from the curves the values of x_1 and x_2 corresponding to the two chosen response values ; the value of x_1 corresponding to the lower frequency is, of course, negative.

The mid-frequency may be computed from

$$\omega_0{}^2 = \omega_1 \omega_2 \frac{x_2 \omega_1 - x_1 \omega_2}{x_2 \omega_2 - x_1 \omega_1} \tag{19}$$

and the ratio Q_1/Q_2 from equation (10), or (17) for 2 or 3 stages or

$$(Q_1/Q_2) = 3(n-1) + \sqrt{3(3n-4)} \text{ for 4 stages.}$$

The Q's of the various stages are then given by :

$$1 \text{ stage :} \quad Q = \frac{n x_1}{\dfrac{\omega_1}{\omega_0} - \dfrac{\omega_0}{\omega_1}} \tag{20}$$

$$2 \text{ stages :} \quad Q_2 = \sqrt{\frac{n Q_2}{Q_1} \cdot \frac{x_1}{\dfrac{\omega_1}{\omega_0} - \dfrac{\omega_0}{\omega_1}}} \;;\; Q_1 = Q_2 \cdot \frac{Q_1}{Q_2} \tag{21}$$

$$3 \text{ stages :} \quad Q_2 = Q_3 = \sqrt[3]{\frac{n Q_2}{Q_1} \cdot \frac{x_1}{\dfrac{\omega_1}{\omega_0} - \dfrac{\omega_0}{\omega_1}}} \;;\; Q_1 = Q_2 \cdot \frac{Q_1}{Q_2} \tag{22}$$

4 stages : $Q_2 = Q_3 = Q_4 = \sqrt[4]{\dfrac{nQ_2}{Q_1} \cdot \dfrac{x_1}{\dfrac{\omega_1}{\omega_0} - \dfrac{\omega_0}{\omega_1}}} \; ; \; Q_1 = Q_2 \cdot \dfrac{Q_1}{Q_2}$ (23)

These amplifiers are made up of one rather selective stage, corresponding to Q_1, and a number of equal broadly-tuned stages, corresponding to $Q_2, Q_3 \ldots$ With increasing feedback the selectivity of Q_1 must be increased, while the selectivity of Q_2 etc. approaches a limiting value which is of the order of the Q of a single stage amplifier without feedback, having the same bandwidth as the complete amplifier with feedback.

The amplifier is first designed, neglecting all capacitances, to have a gain approximately n times the final value. The feedback is designed with a purely resistive feedback network to decrease the gain n times. The value of C_g is then determined at the minimum practicable value, that is the input capacitance of the following stage plus an allowance for strays. Then the values of the other capacitances are given by

$$G_p = \frac{p}{\omega_0 \sqrt{R_L R_g}} \; ; \; C_c = \frac{k}{\omega_0 \sqrt{R_L R_g}} \qquad (24)$$

where $p = \dfrac{b}{2Q}\left[1 - \sqrt{1 + 4Q^2\dfrac{d}{b} - 4Q^2\left(1 + \dfrac{1}{b^2} + d^2\right)}\right]$

$k = \dfrac{1 - pd}{p + d}$; $b = \sqrt{R_g/R_L}$ and $d = C_g \omega_0 \sqrt{R_L R_g}$.

The amplification without feedback at the mid-frequency is

$$\alpha_0 = kQ \cdot g_m \sqrt{R_L R_g} \qquad (25)$$

There are limits to the physically realizable values of C_p and C_c which are quoted in the article (Ref. H13).

It is shown in (C) below that the condition of maximal flatness gives an oscillatory transient response ; for instance when the input voltage is a unit step, the two-stage amplifier has a response which overshoots 4.3%, while the three-stage amplifier overshoots 8% (Ref. H15).

A maximal flatness 2 stage amplifier has a small rise in the response at some high frequency due to positive feedback, compared with the response without feedback. For example, as may be shown by drawing a Nyquist diagram, a 2-stage amplifier with $\alpha_2/\alpha_1 = 10$, $A_0 = 316$, $\beta = 0.016$ has a rise of about 1.4 db due to positive feedback at some high frequency.

(B) Method of C.F. Brockelsby (Ref. H12)

A different approach to the same goal is made by Brockelsby who provides " staggering " by changing the values of load resistance and thereby obtaining different values of gain and frequency response. This method was published earlier than that of Mayr but is less comprehensive, although providing useful additional information. A summary of this method is given below. See also Ref. J13.

Two-stage amplifiers and maximal flatness

The widest frequency range possible without any peaks whatever is known as the condition of " maximal flatness." With a 2 stage amplifier in which both stages have identical gain and frequency response, the condition of maximal flatness is that $\beta A_0 = -1$, corresponding to a gain reduction of 2 times. On the other hand if two r.c.c. pentode stages have " staggered " gain and high frequency response (by using different values of load resistance) the condition of maximal flatness at the high frequency end of the range is that

$$\tfrac{1}{2}(b + 1/b)^2 = 1 - \beta A_0 \qquad (26)$$

where A_0 = amplification at mid-frequency without feedback,
b = factor by which the normal value of load resistance is multiplied for one stage,

$1/b$ = factor by which the normal value of load resistance is multiplied for the other stage,

and β = fraction of the output voltage fed back to the input.

Equation (26) may be expressed in the alternative form

$$F = 2S^2 \tag{26a}$$

where $F = 1 - \beta A_0$ = feedback factor

and $S = \frac{1}{2}(b + 1/b)$ = " staggering coefficient."

In a two stage amplifier with a high frequency peak, the required staggering to produce a peak of known value is given by

$$S^2 = (F/2)(1 - \sqrt{1 - \alpha^2}) \tag{27}$$

where α = ratio of voltage gain at middle frequencies to that at the peak (α is less than 1).

The results are summarized in the following table :

$b =$	1	1.5	2	3	4	5
$S =$	1	1.08	1.25	1.7	2.1	2.6

Feedback factor (F) for maximal flatness (Eqn. 26a) :

$F =$	2	2.3	3.1	5.8	8.8	13.5

For 1 db rise (Eqn. 27) :—

$F =$	3.7	4.3	5.7	10.6	16	25

For 3 db rise (Eqn. 27) :—

$F =$	6.8	8.0	10.7	19.7	30	46

Fig. 7.76 shows the frequency characteristics obtained with 2 stage amplifiers with maximal flatness and for several other conditions. Here

$x = \omega CR$

and $F = 1 - \beta A_0$ = feedback factor.

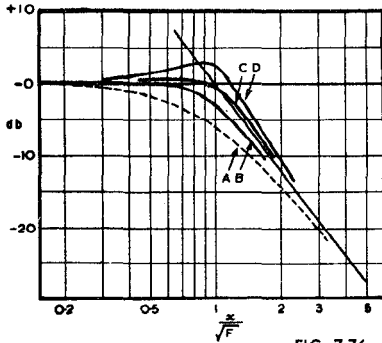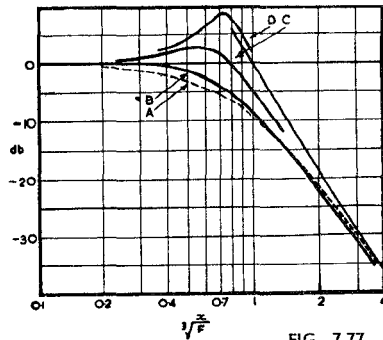
FIG. 7.76   FIG. 7.77

Fig. 7.76. *Frequency characteristics of two-stage amplifier. Curve A is without feedback and curves B, C and D respectively for maximal flatness, a 1 db peak and a 3 db peak ($x = \omega CR$; C = normal shunt capacitance of each stage ; R = total normal a.c. load resistance for each stage ; pentode valves (Ref. H12).*

Fig. 7.77. *Frequency characteristics of three-stage amplifier. Curve A is without feedback and curves B, C, D are respectively for maximal flatness and with feedback equal to two and four times the m.f. value (Ref. H12).*

If the condition of maximal flatness is to hold at the low as well as at the high frequency end of the band, the centre-frequencies of both stages should be identical (Ref. H13).

Three-stage amplifiers and maximal flatness

It has been shown (Ref. H12) that if a substantially flat response over the maximum possible frequency band is required, the optimum arrangement for a three-stage amplifier is to have two stages with wide frequency response and one with narrow.

This finding is based on the assumption that the amplifier has maximal flatness ; it is not necessarily the most desirable arrangement under all conditions.*

The condition for maximal flatness is that

$$F = \tfrac{1}{2} \cdot \frac{(B^3 + 2)^2}{2B^3 + 1} \tag{28}$$

where F = feedback factor
      B = staggering factor = 1/b

and b = factor by which the normal value of load resistance is multiplied for the one (narrow) stage.

[B is always equal to or greater than unity.]

Equation (28) may be approximated within 1.4% when $B \geqq 2$ by

$$F \approx 1 + (B^3/4) \tag{29}$$

For example when $F = 10$ (i.e. 20 db feedback), $B \approx 3.3$ and the ratio of band-widths is approximately 11.

As with the two-stage amplifier, an increased amount of feedback beyond that to give maximal flatness gives high frequency peaks (Fig. 7.77).

The critical amount of feedback to provide self-oscillation is

$$F_c = 5 + 2B^3 + 2/B^3 = 8F_f - 3 \tag{30}$$

where F_c = feedback factor to give critical feedback
and F_f = feedback factor to give maximal flatness.

The stability margin of a three-stage maximal-flatness amplifier is ($F_c - F_f$) and in a typical example

$$F_f = 3.16 \text{ (i.e. 10 db reduction in gain) for maximal flatness}$$

then $F_c = 8 \times 3.16 - 3 = 22.2$ (i.e. 27 db approx.) to give self-oscillation. The stability margin is therefore $27 - 10 = 17$ db approximately.

If the feedback is made very large, and the staggering is adjusted so as always to give maximal flatness, $F_c \approx 8F_f$ which for this example becomes $8 \times 3.16 = 25.2$ (i.e. 28 db approx.) giving a stability margin of 18 db approximately.
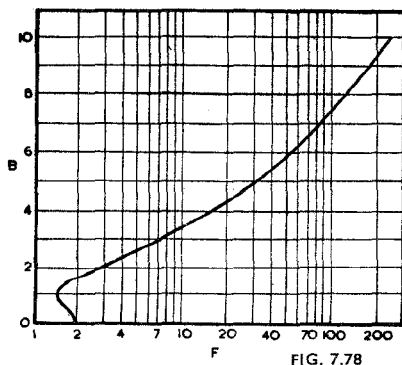

Fig. 7.78. *Relation between the staggering factor B and the gain reduction factor F for maximal flatness in a three-stage amplifier. Values of B less than unity apply to the " two narrow, one wide " condition ; values of B greater than unity apply to the " two wide, one narrow " condition (Ref. H12).*

FIG. 7.78

For both two- and three-stage maximal-flatness amplifiers (high frequency response)

1. The feedback which is fully effective in reducing distortion begins to fall at a frequency about F times lower than the knee.

2. The effective gain-reduction factor is about unity, indicating no feedback, at the knee.

3. The knee of the curve for the two-stage amplifier (Fig. 7.76) is at $x = \omega CR = \sqrt{F}$; at this point the attenuation is 3 db. In the three-stage amplifier (Fig. 7.77) the knee of the curve is at $x = \omega CR = \sqrt[3]{F}$; the attenuation is 3 db at $x = (2/3)^3 \sqrt{F}$.

*The arrangement to provide the greatest value of the product (effective feedback) × (bandwidth) is that due to Duerdoth described in Sect. 3(v)E.

Low frequency response

The equations for the high frequency end may be applied to the low frequency end of the response characteristic, in the simple case when the only frequency variable is the grid coupling condenser, by giving x the value

$$x = 1/\omega C_g R_g \tag{31}$$

where C_g = grid coupling capacitance
and R_g = grid resistor plus the effective source impedance of the preceding stage
              (approximately R_L in the case of a pentode).

(C) Conditions for critical damping

Critical damping may be defined as that value which gives the most rapid transient response which is possible without overshoot.

The maximal flatness amplifier—see (A) and (B) above—has been examined by J. E. Flood (Ref. H15) in the light of critical damping; the following summary is based on his work. It has been shown that a two-stage maximally-flat amplifier has a response which overshoots 4.3% while the three-stage amplifier overshoots 8% when a step waveform is applied to the grid circuit.

Although this article is concerned mainly with video-frequency or pulse amplification, it is possible that critical damping is a desirable feature in a-f amplifiers and that a condition of under-damping with oscillatory response to transients is to be avoided. See general comments in Sect. 3(v), also Ref. J8.
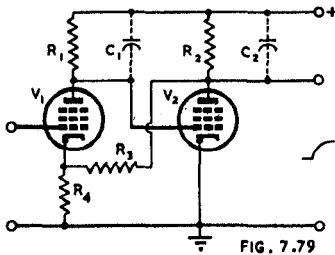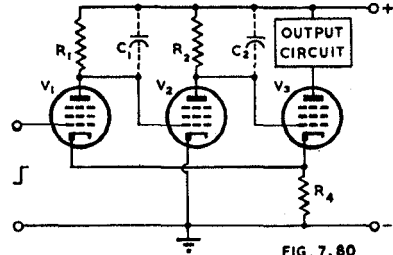

Fig. 7.79. *Two-stage uncompensated amplifier with feedback (Ref. H15).*
Fig. 7.80. *Three-stage uncompensated amplifier with two time constants within the feedback loop (Ref. H15).*

Two stage amplifier with constant value of β (Fig. 7.79)

The following may also be applied to the three stage amplifier of Fig. 7.80 which has only two time constants within the feedback loop.

It may be shown that the condition for critical damping is that

$$1 - \beta A_0 = \frac{(\alpha_1 + \alpha_2)^2}{4\alpha_1\alpha_2} \tag{32}$$

where A_0 = amplification at the mid-frequency without feedback
      β is negative and independent of frequency
      α_1 = $1/R_1C_1$ = inverse time constant of V_1
and α_2 = $1/R_2C_2$ = inverse time constant of V_2.

When a higher feedback factor than that indicated by eqn. (32) is used, the transient response will be oscillatory; when a lower feedback factor is used, the transient response will be over-damped.

Equation (32) may be put into the form

$$1 - \beta A_0 = (1 + K)^2/4K = K/4 + 1/2 + 1/4K \tag{33}$$

where $K = \alpha_2/\alpha_1$ = ratio of inverse time constants.

This is plotted in the lower curve of Fig. 7.81.

If a fairly large amount of feedback is to be used, the ratio of time constants must be large.

The resultant inverse time constant of the amplifier is given by

$$\alpha = \tfrac{1}{2}(\alpha_1 + \alpha_2) = \tfrac{1}{2}\alpha_1(1 + K) \tag{34}$$

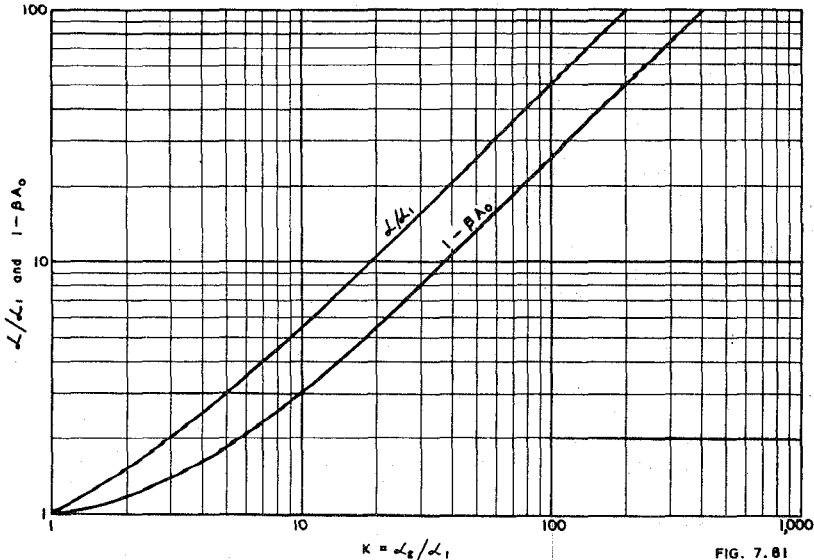This relationship is plotted in the upper curve of Fig. 7.81.

Fig. 7.81.   *Conditions for critical damping of two-stage uncompensated amplifier*
(Ref. H15).

Note : The " staggering coefficient " S as used by Brockelsby (Ref. H12) is related
to α_1 and α_2 by the equation

$$S = \frac{\alpha_1 + \alpha_2}{2\sqrt{\alpha_1 \alpha_2}} = \tfrac{1}{2}\left(\sqrt{K} + \frac{1}{\sqrt{K}}\right) \tag{35}$$

The condition for maximal flatness is therefore given by

$$1 - \beta A_0 = \frac{(\alpha_1 + \alpha_2)^2}{2\alpha_1 \alpha_2} \tag{36}$$

Comparing equations (32) and (36), it will be seen that for any particular ratio of
α_2/α_1, the condition of critical damping permits a feedback factor of only one half
that required for maximal flatness of the frequency characteristic.   The **transient
response of the maximally-flat amplifier is therefore oscillatory**.

A 2-stage critically damped amplifier will normally have a small amount of positive
feedback at high frequencies, although the rise in response in a typical case will be
less than 1 db above the zero-feedback curve.

Two-stage compensated amplifier

If resistor R_3 in Fig. 7.79 is shunted by a capacitor C_3 and if R_3 is very much greater
than R_4 then critical damping is obtained when the inverse time constant of the feed-
back path is given by

$$\alpha_3 = \frac{-\alpha_1 \alpha_2 \beta A_0}{2\sqrt{\alpha_1 \alpha_2 (1 - \beta A_0)} - (\alpha_1 + \alpha_2)} \tag{37}$$

where $\alpha_1 = 1/R_1 C_1$; $\alpha_2 = 1/R_2 C_2$; $\alpha_3 = 1/R_3 C_3$
and β is negative for negative feedback.

Similarly if an inductance L is connected in series with R_4 in Fig. 7.80, and if $g_m R_4$
is very much less than 1, then critical damping is obtained as indicated by eqn. (37)
where $\alpha_3 = R_4/L$.

The inverse time constant for the amplifier is given by

$$\alpha = \sqrt{\alpha_1 \alpha_2 (1 - \beta A_0)} \tag{38}$$

Using the value of α_3 given by eqn. (37) for the inverse time constant of the amplifier
with feedback, critical damping is obtained and the response to a step waveform of the

feedback amplifier is made identical with that of an amplifier without feedback having two stages, each having an inverse time constant equal to the geometrical mean of the inverse time constants of the two stages multiplied by the square root of the feedback factor (eqn. 38).

A particularly simple special case is obtained when

$$\alpha_2 = \alpha_1(1 - \beta A_0) ;$$

then $\alpha = \alpha_3 = \alpha_2 = \alpha_1(1 - \beta A_0)$.

Values of α_3 for critical damping may be derived from the curves of Fig. 7.82 and values of α from Fig. 7.83.

Three-stage amplifier

Critical damping cannot be obtained in a three-stage amplifier with constant β.

Three-stage amplifier with single time constant in feedback path

In this case there is only one value of feedback factor as given by eqn. (39) for which critical damping can be obtained.

$$1 - \beta A_0 = \frac{(\alpha_1 + \alpha_2 + \alpha_3)^2}{27\alpha_1\alpha_2\alpha_3} \tag{39}$$

$$\alpha = (\alpha_1 + \alpha_2 + \alpha_3)/3 \tag{40}$$

$$= [(\alpha_1\alpha_2\alpha_3(1 - \beta A_0)]^{1/3} \tag{41}$$

$$\alpha_4 = \frac{- 3\alpha_1\alpha_2\alpha_3\beta A_0}{\alpha_1{}^2 + \alpha_2{}^2 + \alpha_3{}^2 - (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)} \tag{42}$$

where α_3 = inverse time constant of third stage

α_4 = inverse time constant of feedback path
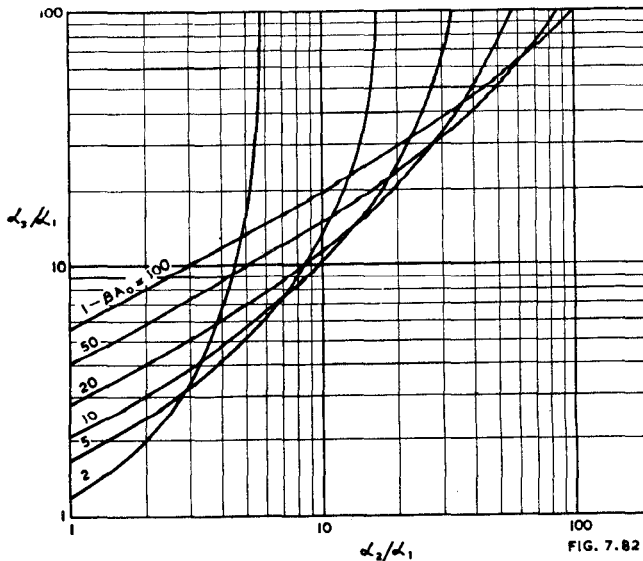
and α = inverse time constant of amplifier.

FIG. 7.82

Fig. 7.82.   *Curves for determining α_3 for critical damping of two-stage compensated amplifier (Ref. H15).*
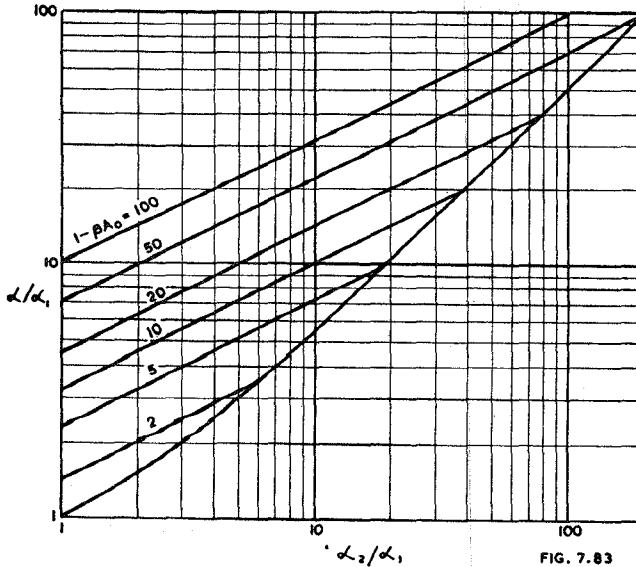
FIG. 7.83

Fig. 7.83.   Curves for determining α for critical damping of two-stage compensated amplifier (Ref. H15).

Three-stage amplifier with two time constants in feedback path

Critical damping can be obtained provided that

$$1 - \beta A_0 > \frac{(\alpha_1 + \alpha_2 + \alpha_3)^3}{27\alpha_1\alpha_2\alpha_3} \tag{43}$$

Under this condition,

$$\alpha_4 = \alpha_5 = (b \mp \sqrt{b^2 - 4ac})/2a \tag{44}$$

$$\alpha^3 = \alpha_1\alpha_2\alpha_3(1 - \beta A_0) \tag{45}$$

where α = inverse time constant of amplifier

$\alpha_1, \alpha_2, \alpha_3$ = inverse time constants of three stages

α_4, α_5 = inverse time constants in feedback path

$a = 3\alpha - (\alpha_1 + \alpha_2 + \alpha_3)$

$b = 3\alpha^2 - (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)$

and $c = \alpha^3 - \alpha_1\alpha_2\alpha_3$.

When $1 - \beta A_0$ is less than the right hand side of eqn. (43) the transient response is over damped and less rapid.

A particularly simple case is obtained when

$$\alpha_2 = \alpha_3 = \alpha_1(1 - \beta A_0)$$

then $\alpha = \alpha_5 = \alpha_4 = \alpha_3 = \alpha_2 = \alpha_1(1 - \beta A_0)$ \hfill (46)

(viii) Constancy of characteristics with feedback

This is sometimes called " stability " (Ref. H8) but refers to the more or less complete independence of the gain of an amplifier with feedback (inside the amplifier frequency range) with regard to changes in valve characteristics and supply voltages. Becker defines the " stability factor " as

$$SF = 1 + \beta A_0 X \tag{47}$$

where $A_0 X$ = lowest gain which the amplifier ever has under the worst operating conditions, without feedback

and SF = stability factor with negative feedback.

(ix) Effect of feedback on phase shift*

The amplification without feedback may be expressed in the complex form

$$A = a + jb = |A| \angle \theta \tag{48}$$

where $|A|$ is the magnitude of the voltage amplification and θ is the phase angle of the output voltage of the amplifier relative to the input voltage, without feedback.

It is obvious that $\theta = \tan^{-1}(b/a)$ and $|A| = \sqrt{a^2 + b^2}$.

The amplification with feedback may be expressed in the form

$$A' = \frac{A}{1 - \beta A} = \frac{a + jb}{1 - (a + jb)\beta} = \frac{a - (a^2 + b^2)\beta + jb}{(1 - a\beta)^2 + b^2\beta^2} \tag{49}$$

provided that there is no phase shift in the feedback network. The phase angle of the output voltage relative to the input voltage, with feedback, is given by

$$\alpha = \tan^{-1} \frac{b}{a - (a^2 + b^2)\beta} = \tan^{-1} \frac{b/a}{1 - \beta A\sqrt{1 + b^2/a^2}} \tag{50}$$

$$= \tan^{-1} \frac{\tan \theta}{1 - \beta A\sqrt{1 + \tan^2 \theta}}. \tag{51}$$

When β is negative, as is the case when the feedback is negative, it is evident from (51) that the angle α is less than the angle θ. Hence the phase shift of the output voltage relative to the input voltage is reduced by negative feedback.

SECTION 4 : SPECIAL APPLICATIONS OF FEEDBACK

There are so many applications of feedback, many of which are outside the subject of radio receiver and amplifier design, that it is impossible even to list them here. The following selected applications have some interesting features.

(1) Electronic attenuators

A cathode follower may be used with various cathode load resistors selected by a rotary switch, to give the coarse steps, and then the screen voltage may be adjusted to give a fine adjustment of attenuation.

Smith, F. W., and M. C. Thienpont " Electronic attenuators " Comm. 27.5 (May 1947) 20.

(2) Stable admittance neutralization

Two valves in cascade with negative voltage feedback may be used to neutralize the input admittance of an amplifying valve (mainly the result of its input capacitance) and thereby enable the use of a load resistance of 23 megohms with a phototube for a cut-off frequency of 20 000 c/s.

" Stable admittance neutralization " Electronic Eng. 14.167 (Jan. 1942) 594.

(3) An amplifier without phase distortion

O. H. Schade has developed an amplifier without cathode by-pass condensers, screen by-pass condensers, decoupling condensers or output condenser in B-supply filter, which has negligible phase distortion from 10 to 30 000 c/s.

" An amplifier without phase distortion," Elect. 10.6 (June 1937) 26:

(4) Stabilized negative impedances

Negative impedance is produced from a positive impedance of the desired type by positive feedback through an amplifier stabilized with negative feedback.

Ginzton, E. L. " Stabilized negative impedances " Elect. (1) 18.7 (July 1945) 140 ; (2) 18.8 (Aug. 1945) 138 ; (3) 18.9 (Sept. 1945) 140.

(5) Tone Control

This is covered in detail in Chapter 15.

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*Method after H. J. Reich " Theory and application of electron tubes."

SECTION 5 : VALVE CHARACTERISTICS AND FEEDBACK

(i) Triode cathode follower   (ii) Pentode cathode follower   (iii) Triode with voltage feedback   (iv) Pentode with voltage feedback, transformer-coupled   (v) Cathode degenerative triode   (vi) Cathode degenerative pentode   (vii) Cathode-coupled triodes   (viii) Feedback over two stages.

Although it is important to bear in mind that the application of feedback does not alter the characteristics of a valve, yet there are certain features which can best be visualized by the graphical use of effective valve characteristics with feedback.

(i) Triode cathode follower

The cathode-follower characteristics (Fig. 7.85) may be drawn very readily from the published triode plate characteristics (Ref. D2). The $E_c = 0$ curve remains as the boundary of the grid current region. If the grid circuit impedance is high, it is not practicable to go beyond the $E_c = 0$ curve, just as with plate-loading.
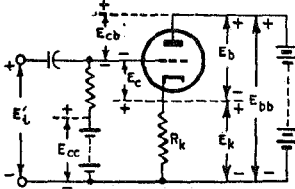
FIG. 7.84

Fig. 7.84. Basic circuit of cathode follower.

The basic circuit is shown in Fig. 7.84 from which it is evident that with zero input voltage $(E_i' = 0)$

$$E_{cb} = E_b - E_c \qquad (1)$$

The plate is the most convenient electrode to regard as the common basis for electrode voltages, since it is the only one directly connected to the voltage source. Valve characteristics may be plotted by the method described below so that each curve, instead of applying to constant grid-to-cathode voltage applies to constant grid-to-plate voltage (E_{cb}). In effect, the valve and its load resistor are inverted as compared with normal practice and the cathode here takes the place of the plate.

Applying eqn. (1) for the case where E_{cb} is constant,

$$E_b = E_{cb} + E_c \qquad (2)$$

For example, take the case when $E_{cb} = 150$ volts. We may calculate the points along the curve using eqn. (2) :

$E_{cb} =$	150	150	150	150	150	volts
$E_c =$	0	−2	−4	−6	−8	volts
$E_b =$	150	148	146	144	142	volts

These points may be plotted to give the $E_{cb} = 150$ volt curve on the existing " plate characteristics," as has been done in Fig. 7.85.

(A) Resistance loaded triode cathode follower

The cathode-follower characteristics may then be used as any ordinary resistance-loaded triode for the determination of power output, voltage gain and distortion. Maximum power output will generally be achieved when R_k is between r_p and $3r_p$, with slightly less distortion with the higher values of R_k. In Fig. 7.85, R_k is taken as 15 000 ohms, with $E_{bb} = 200$ volts. If it were desired to obtain the highest possible voltage swing, a much higher value of load resistance would be selected (e.g. 50 000 ohms). In each case the operating point should be selected so as to avoid both grid current and plate-current cut-off with the highest input voltage. For maximum output, a point slightly above half-way along the loadline is desirable. In Fig. 7.85, a suitable operating point (O) is the intersection of the $E_{cb} = 150$ volt curve with the loadline which occurs at a plate-to-cathode voltage of 145 volts, thus indicating that $E_c = -5$ volts. Alternatively, the bias could have been read from intersection of the $E_c = -5$ (plate characteristic) with the loadline.

When the operating point (O) has been established, it is then necessary to work entirely with the E_{cb} curves, neglecting the E_c curves. In Fig. 7.85 the operating point is at $E_{cb} = 150$ volts, and point A is $E_{cb} = 100$, so that the peak input voltage
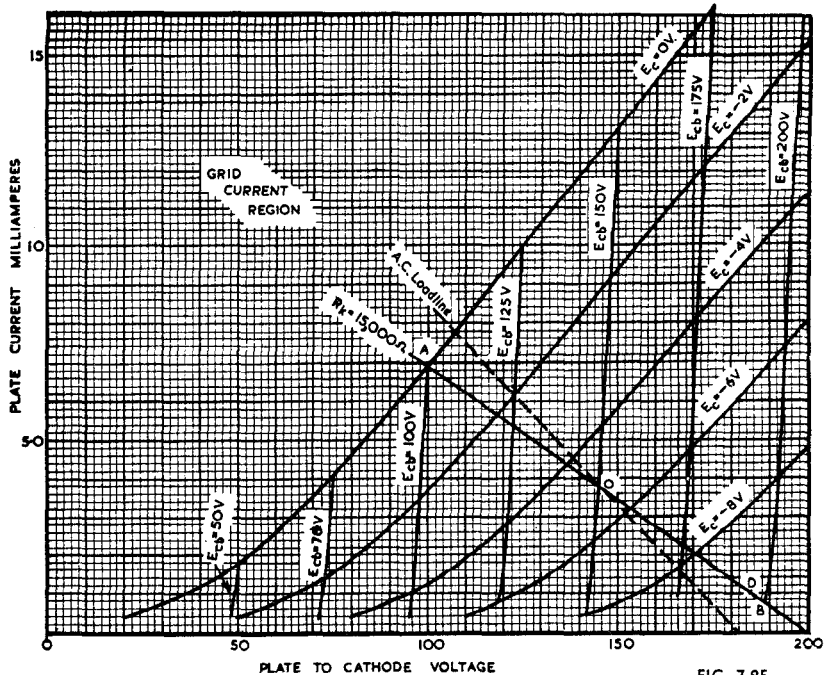
FIG. 7.85

Fig. 7.85. Cathode follower characteristics of small general purpose triode. The solid loadline is for $R_k = 15\,000$ ohms, the dashed loadline is for the addition of an a.c. shunt load.

amplitude is 50 volts. It is obvious that the opposite swing will extend to $E_{cb} = 200$ volts (point B) thus giving a peak-to-peak swing of $2 \times 50 = 100$ volts. It will be seen that OA corresponds to $E_c = -5$ volts, while OB corresponds to -5.7 volts, but this is due to the improved linearity with the cathode follower. With plate loading the loadline would be AD, so that cathode loading causes a longer loadline by the amount DB, thus resulting in a slightly greater power output.

The voltage gain is given by

$$A' = \frac{E_B - E_A}{E_{cb(B)} - E_{cb(A)}} \tag{3}$$

where subscripts A and B indicate the voltages at points A and B on the loadline.

If the cathode is coupled through a coupling condenser C_c to a following grid resistor R_g (Fig. 7.8), the a.c. loadline will be drawn through the operating point O with a slope of $-1/R_L$ where $R_L = R_k R_g/(R_k + R_g)$.

The mutual characteristics of cathode followers may be drawn and interpreted by the method of Ref. C29.

(B) Transformer-coupled triode cathode follower

The same form of cathode-follower characteristic may be used when the load is coupled through a transformer so as to reflect a resistance R_k across the primary. In most cases the load resistance may be taken (at least as a first approximation) to have the same value as for plate loading. In Fig. 7.86 the loadline AOB extends from A (the junction of the $E_c = 0$ and the $E_{cb} = 100$ V curves) to B, on the $E_{cb} = 500$ V curve. The operating point O is the intersection of the loadline with the $E_{cb} = 300$ V curve, thus giving a peak input voltage of 200 volts in both directions. The power output and distortion may be calculated as for plate loading.

If the load resistance R_k is increased, the loadline will rotate about O to the position (say) CD with constant input voltage. This will usually result in a decrease in both

power output and distortion—there is the same advantage (as with plate loading) in keeping the loadline out of the region of bottom curvature. It would be possible, with the higher resistance load, to increase the input voltage so as to extend the loadline from OC to meet the $E_c = 0$ curve.

If the load resistance R_L is decreased, the loadline will rotate about O to the position (say) EF. If the input voltage is unaltered, the plate current will cut off at F and the valve will run into grid current at E, both resulting in distortion. In this, the cathode follower differs from a plate loaded triode, and it is important to ensure that the load resistance does not fall appreciably below the designed value. Fortunately, a loudspeaker has an impedance characteristic which, although it rises considerably above its nominal (400 c/s) value, does not drop appreciably below it. A cathode follower with a loudspeaker load is therefore a good combination from the loading point of view.
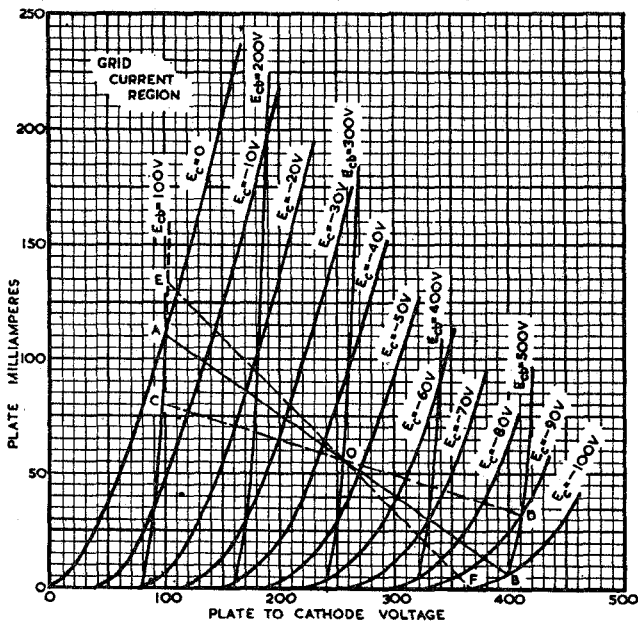

FIG. 7.86

Fig. 7.86. *Cathode follower characteristics of power triode (type 2A3) with transformer-coupled load. Loadline AB is normal ; CD is high resistance load line ; EF is low resistance loadline illustrating grid current and plate current cut-off.*

If a low impedance driver is used, it is possible to drive a cathode follower into the grid current region with considerably greater power output and efficiency. In the ideal limiting case, the conditions are :

Plate voltage	E_b volts
Plate current	I_b amperes
Load resistance	E_b/I_b ohms
Power output	$\frac{1}{2}E_bI_b$ watts
Plate efficiency	50%

If the cathode follower is to be used as a high-input-impedance voltage amplifier, the conditions vary with the output level. At low output voltages, the operating conditions are of no consequence (within reasonable limits) so far as voltage gain is concerned, and R_k may be equal to r_p. For very high output voltages, the load resistance should be as high as practicable (assuming a purely resistive load), say $R_k = 5r_p$, and the operating current should be as low as practicable without causing plate-

current cut-off. A somewhat lower load resistance may be used for fairly high output voltages.

The transformer primary inductance should have the same value as for plate loading under similar conditions, if full power output is desired at low frequencies. A transformer with low primary inductance will give uniform gain at low level output, but at high output it will cause plate-current cut-off and grid current as for loadline EF in Fig. 7.86.

References C14, C16, C17, C29, D1, D2, D3, D8, D9, D10, D11.

(ii) Pentode cathode follower

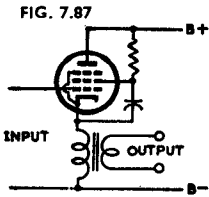A pentode (or tetrode) may be connected in several ways arising from the screen supply and by-pass—

FIG. 7.87

Fig. 7.87. Pentode cathode follower, transformer-coupled, screen by-passed to cathode.

(A) Triode connection (screen tied to plate)

The published "triode" characteristics may generally be used. If no triode characteristics are available, ascertain the characteristics for the screen voltage corresponding to the desired conditions (the plate voltage may be equal or higher).

Then $I_k \approx I_b + I_{c2}$ (4)

$g_m \approx (\text{pentode } g_m) \times (I_k/I_b)$ (5)

and $\mu \approx E_{c2}/E_{c1}$ (cut-off).   (6)

(B) Screen by-passed to earth

There is no exact method using published characteristics. An approximate method has been described by Shapiro (Ref. D2).

(C) Screen by-passed to cathode—Transformer-coupled (Fig. 7.87)

The cathode follower characteristics may be drawn by the same method as for a triode (Fig. 7.88). The procedure for calculating the power output and distortion is :
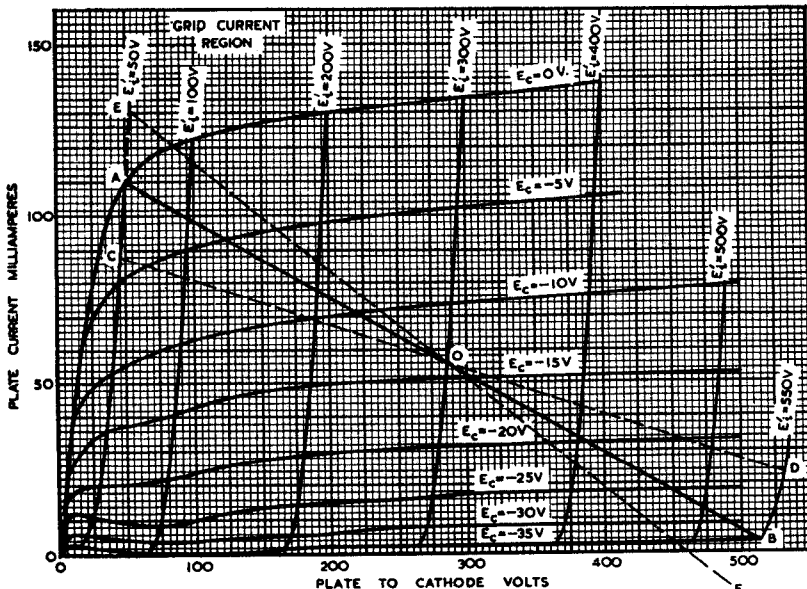

FIG. 7.88

Fig. 7.88. Cathode follower characteristics of beam power tetrode and typical of all pentodes. Loadline AOB is normal ; CD high resistance ; EF low resistance illustrating grid current and plate current cut-off.

1. To determine the maximum plate voltage and current for the operating point. Here E_b = 285 V and happens to coincide with the E_{cb} = 300 V curve. If desired, mark the value of E_i' corresponding to point O.

2. Try several values of load resistance (i.e. slope of loadline) until an optimum position for point A is determined, so as to make OA as long as possible, and at the same time give OB = OA with B slightly above plate-current cut-off.

3. Determine E_b for point A (in this case 50 V)—A is then automatically on the E_{cb} = 50 V curve.

4. Draw the other limiting E_{cb} curve (in this case E_{cb} = 550 V, giving a peak amplitude of 250 V each way).

5. Calculate power output and distortion as for a power triode, using the loadline AOB and the E_{cb} curves.

When a higher resistance load is used with the same input voltage, the loadline rotates about O to (say) COD, where the power output is lower, the distortion slightly less, and point C is well negative with respect to E_c = 0. When a lower resistance load is used with the same input voltage, the new loadline may be (say) EOF where E is well into the grid-current region and F is beyond plate-current cut-off. In this respect the pentode is similar to the triode, but the pentode is rather more critical regarding low load resistances ; it is, however, quite satisfactory with a speaker load.

If cathode bias is used, the procedure is as for a plate-loaded power pentode.

(D) **Screen by-passed to cathode—Resistance-loaded** (Fig. 7.89)

The curves are generally the same as for transformer coupling, and the treatment is similar to that for resistance-loaded triodes, except that it is necessary to allow for the d.c. screen current flowing through R_k. The screen current may be taken from the published data for typical operating conditions, or estimated from the ratio of plate to screen currents. The voltage drop through R_k additional to that due to the plate current will be $R_k I_{c2}$. The procedure is to take as the effective plate supply voltage the value $(E_{bb} - R_k I_{c2})$ and then to proceed normally, as for a triode.

(E) **Screen voltage from separate supply** (Fig. 7.90)

If the screen voltage is obtained from a separate supply, such as a battery, returned to the cathode, then the voltage from cathode to screen may be maintained constant without smoothing out the screen current variations. In this case the screen current does not flow through the cathode resistor R_k. The pentode characteristics for a constant screen voltage may be used without any adjustment for the effects of screen current.
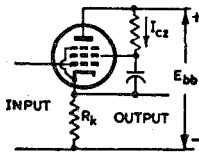
References D1, D2, D3, D11.

FIG. 7.89   FIG. 7.90

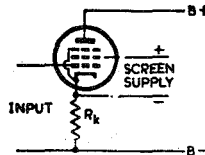*Fig. 7.89. Pentode cathode follower, resistance-loaded, screen by-passed to cathode.*   *Fig. 7.90. Pentode cathode follower, resistance-loaded screen from separate supply.*

(iii) Triode with voltage feedback

The basic circuit is Fig. 7.91 in which we can consider the conditions with direct applied voltages (E_i' and E_b). A fraction of the plate voltage is applied degeneratively to the grid so that

$$|\beta| = R_1/(R_1 + R_2) \tag{7}$$

where $|\beta|$ is the numerical value of β without regard to its sign.

Consider first the condition for E_c to be zero. It is evident that the condition is that

$$E_i' = |\beta| E_{bo} \tag{8}$$

where E_{bo} = value of E_b when E_c = 0.

Now consider the general condition when the grid is negative with respect to the cathode. If the input voltage E_i' is kept constant, then the grid-to-cathode voltage E_c will change when E_b is changed. The relationship is

$$\Delta E_e = |\beta|(\Delta E_b) \qquad \text{when } E_i' \text{ is constant.}$$

Therefore $\Delta E_b = \Delta E_c/|\beta|$

Therefore $E_b = E_{b0} - \Delta E_b = E_{b0} - \Delta E_c/|\beta|$ \hfill (9)

Eqn. (9) may be used for the calculation of points on the constant E_i' characteristics, as for example :

E_{b0}	=	100	100	100	100	100 volts
$|\beta|$	=	0.2	0.2	0.2	0.2	0.2
E_i'	=	20	20	20	20	20 volts
$-\Delta E_c$	=	0	-1	-2	-3	-4 volts
$-\Delta E_c/|\beta|$	=	0	-5	-10	-15	-20 volts
E_b	=	100	95	90	85	80 volts

These have been plotted in Fig. 7.92. It should be noted that E_{b0} is the plate voltage corresponding to zero grid voltage.

(iv) Pentode with voltage feedback—Transformer coupled

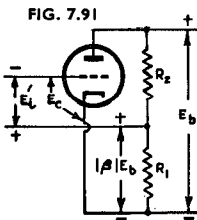Pentodes with a substantial degree of negative voltage feedback (say $|\beta| \geqq 0.2$)

FIG. 7.91

behave more or less like a cathode follower. Pentodes with a limited degree of feedback, or none, exhibit some special peculiarities. Fig. 7.93 shows input voltage curves for three different values of β ; the curvature on the $\beta = -0.1$ curve continues along its whole length while its slope is gradual and bottom curvature extensive.

Fig. 7.91. Basic circuit of triode with negative voltage feedback.

The full effect of a low degree of feedback is illustrated in Fig. 7.94 where $\beta = -0.1$. The curves are drawn as in the triode case except for the change in β. A suitable loadline AOB has been selected to give about the same power output as without feedback. Provided that the load resistance remains constant, the performance is satisfactory, but the value is critical. With constant input voltage (40 volts peak)
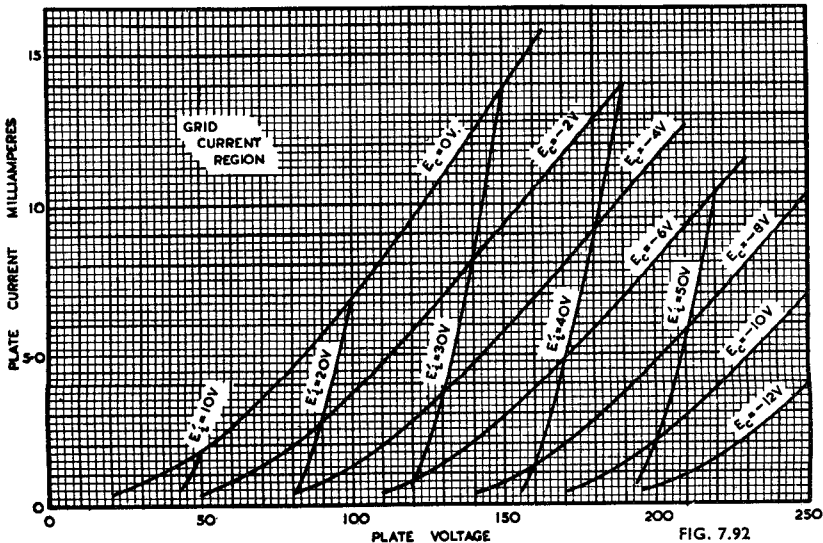
Fig. 7.92. Feedback characteristics of small general purpose triode with 20% negative voltage feedback ($\beta = -0.2$).
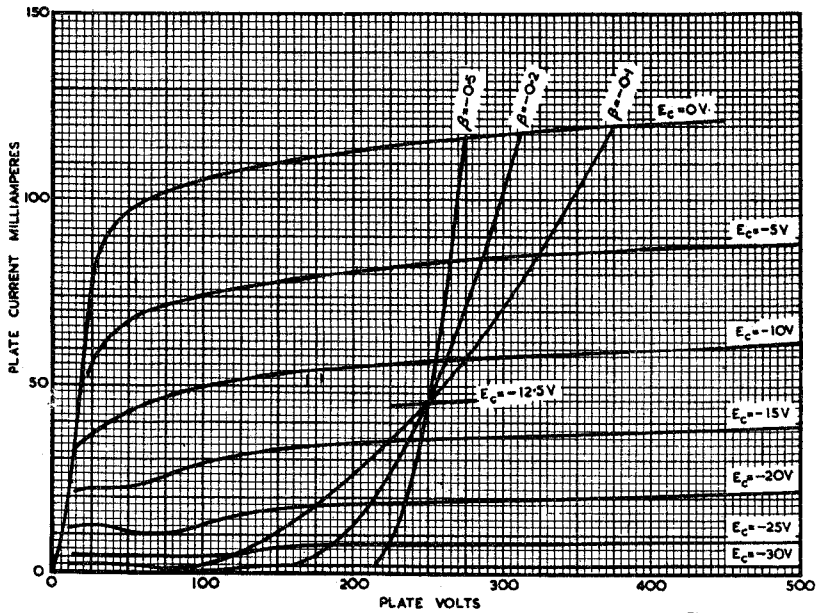
FIG. 7.93

*Fig. 7.93. Single input voltage curve for each of three values of voltage feedback
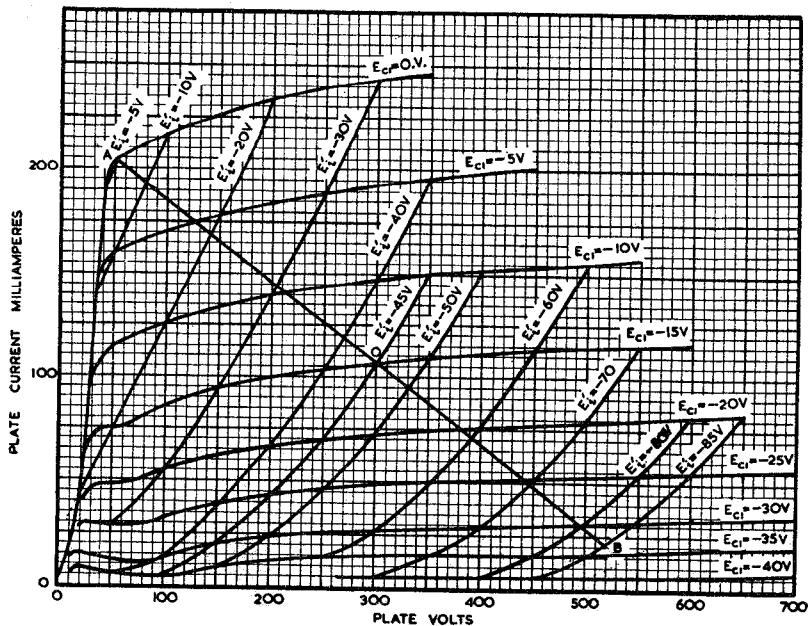for beam power tetrode (6V6-GT with $E_{c2} = 250\ V$), to illustrate effect of feedback.*

FIG. 7.94

*Fig. 7.94. Feedback characteristics of beam power tetrode (6L6 or 807 with
$E_{c2} = 300\ V$) and 10% voltage feedback ($\beta = -0.1$).*

the valve will run into grid current as soon as the load resistance either increases or decreases. It is therefore not suitable for use on a loudspeaker load unless the input voltage is reduced. A 10% reduction in input voltage (19% in power output) would be some improvement, but greater reduction is desirable, say 20% to 30% of the input voltage, depending on the operating conditions. Even with an input reduction giving a power output of only 50% maximum, the pentode is still less flexible with regard to load resistance than an ordinary triode. This position improves as the amount of feedback is increased.
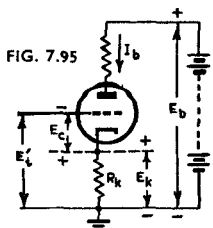
See References D4, D12.

FIG. 7.95

Fig. 7.95. *Basic circuit of cathode degenerative triode.*

(v) Cathode degenerative triode

The circuit diagram of a cathode degenerative triode is shown in Fig. 7.95 from which it is evident that

$$E_c = E_i' - E_k \tag{10}$$

The curves of a cathode degenerative triode may be drawn by the procedure outlined below, although a special set of characteristics is required for each value of R_k. Fig. 7.96 shows the curves for a typical general purpose triode with $R_k = 1000$ ohms. The input voltage curves are straighter than those without feedback, and the two curves coincide only at $I_b = 0$. The E_i' curves have a lower slope, indicating a higher plate resistance, than the E_c curves.

Take the $E_i' = -4$ V curve as an example of the calculations. We know that $R_k = 1000$ ohms and $E_i' = -4$ V. If $I_b = 2$ mA, then $E_k = 2$ V and $E_c = E_i' - E_k = -4 - 2 = -6$ V.

Refer to the plate characteristics to find the plate voltage which will give a plate current of 2 mA at a bias of -6 V —the value is $E_{pk} = 144$ volts.
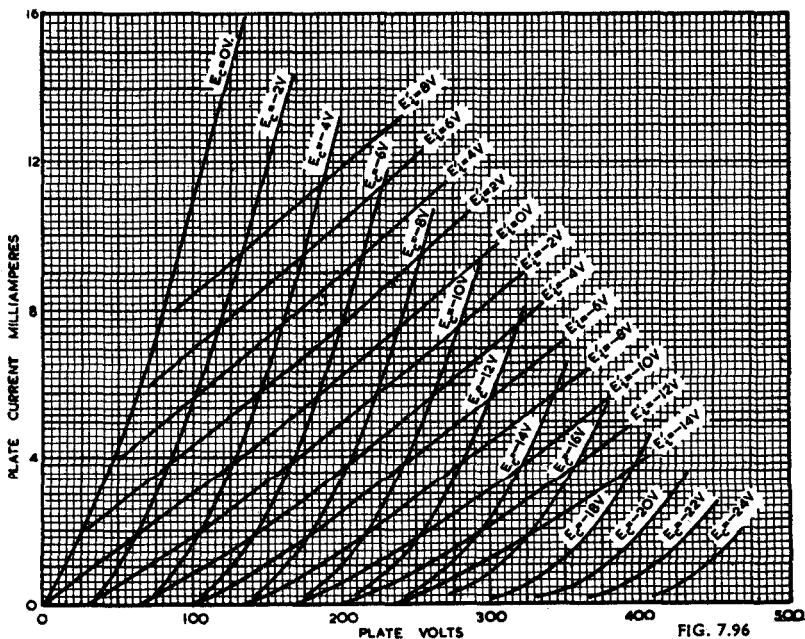

FIG. 7.96

Fig. 7.96. *Current feedback characteristics of general purpose triode (6SN7-GT single unit) with $R_k = 1000$ ohms.*

The total voltage (E_b) from plate to earth is
$$E_b = E_{pk} + E_k = 144 + 2 = 146 \text{ V.}$$
Therefore the point $(E_b = 146 \text{ V}; \ I_b = 2 \text{ mA})$ is on the $E_i' = -4$ V curve. Repeat this procedure for $I_b = 0, 4,$ and 6 mA to give the whole curve; then perform a similar operation for $E_i' = -6$ V, -8 V and so on.

The loadline may then be drawn in the normal manner, except that its slope will be $-1/(R_k + R_L)$. The E_i' curves are to be used for calculating gain and distortion.

The method generally employed with cathode degenerative triodes makes use of the published characteristics. There are several methods including those of Middleton, McIlroy, Huber, Lonsdale and Main; the following treatment is based on Krauss, and may also be applied to cathode followers.

As an example take the characteristics of Fig. 7.97 with $R_k = 8\ 000$ ohms, $R_L = 32\ 000$ ohms and $E_{bb} = 400$ volts. Draw the loadline corresponding to $R_k + R_L = 40\ 000$ ohms, as shown. Add an E_k scale below the E_b scale, based on the equation $E_k = R_k I_b$, commencing from E_{bb}. The value of E_k at any point on the loadline may then be found by projecting downwards to the E_k scale.

The input voltage E_i' is not proportional to any scale on the diagram, because the amplification is not constant. The value of E_i' at any point of intersection along the loadline may be found by drawing up the following table where each value of E_c is taken in turn. The input voltage is actually $(E_k + E_c)$, but for convenience the point $(E_c = -8)$ has been selected as the operating point, and so shown in the E_i' column.

E_c	E_k	$(E_k + E_c)$	E_i'	
0	64	64	34.7	Point A
−2	57	55	25.7	
−4	50.2	46.2	16.9	
−6	43.6	37.6	8.3	
−8	37.3	29.3	0	Operating point (O)
−10	31.2	21.2	−8.1	
−12	25.1	13.1	−16.2	
−14	19.6	5.6	−23.7	
−16	14.7	−1.3	−30.6 ⎱ Point B at $E_k = 11.9$
−18	10.1	−7.9	−37.2 ⎰ and $E_i' = -34.7$
−20	6.2	−13.8	−43.1	

The values of E_i' may be plotted against E_k and therefore also against E_b to give the dynamic characteristic, but this is usually unnecessary. Once the maximum input voltage has been selected it is only necessary to mark the extremities on the loadline. For example, if it is desired to swing to $E_c = 0$, then the peak E_i' will be 34.7 volts in each direction. The peak in one direction will be A where $E_c = 0$, and the other will be B, determined by interpolation:

$$E_c = -16 \qquad E_k = 14.7 \qquad E_i' = -30.6$$
$$E_c = -18 \qquad E_k = 10.1 \qquad E_i' = -37.2$$
$$\overline{\text{Diff.} = \qquad\qquad 6.6}$$

Also the difference between the desired value (-34.7) and -30.6 is 4.1. The ratio is therefore 4.1/6.6.

Similarly with E_k: $14.7 - 10.1 = 4.6$.

The value of E_k for point B is therefore $14.7 - 4.6 (4.1/6.6) = 14.7 - 2.8 = 11.9$.

The voltage gain and power output may be calculated from $E_b = 5E_k$, using the values of E_k at $E_i' = 34.7$ (point A), $E_i' = 0$ (point 0) and $E_i' = -34.7$ (point B). The second harmonic distortion may be found from the ratio AO/OB measured in volts (E_k); i.e. $(64 - 37.3)/(37.3 - 11.9) = 26.7/25.4 = 1.05$. Therefore $H_2 \approx 1\%$—see Chapter 13 Sect. 2(i).
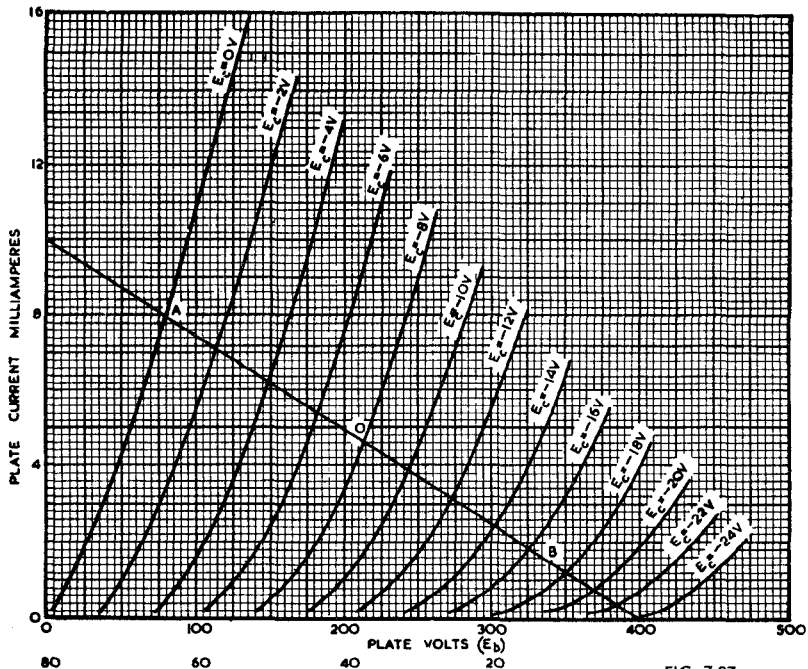
References D1, D3, D5, D8, D9, D10, D11.

Fig. 7.97. Method of calculating performance of cathode degenerative triode without drawing special characteristics.

(vi) Cathode degenerative pentode

The general procedure is the same as for triodes, except that allowance must be made for the d.c. screen current flowing through R_k. This is done by taking $(E_{bb} - R_k I_{c2})$ as the effective plate supply voltage, and then by carrying on as for the triode case.

For valves with a high plate resistance this method is not very satisfactory, and a more practical method has been described by Pratt.

References D1, D11.

(vii) Cathode-coupled triodes

It is possible to make a graphical analysis of the cathode-coupled amplifier, using ordinary published valve characteristics (Ref. D6).

(viii) Feedback over two stages

It is possible to draw the equivalent characteristics of two stages in cascade with feedback over both stages, following the method of Pratt (Ref. D1).

Alternatively, it is possible to use the published valve characteristics to obtain a graphical analysis (Ref. D7).

SECTION 6 : REFERENCES TO FEEDBACK

A) GENERAL REFERENCES TO THEORY OF FEEDBACK
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