OUTPUT TRANSFORMERS PART I*

BY JAMES MOIR

The output tubes in common use in audio amplifiers require anode loads of between about 1kΩ and 10kΩ if the maximum undistorted power output is to be obtained. There are practical difficulties in winding a loudspeaker voice coil with the large number of turns of fine wire required to achieve such high load impedances directly and thus it is common practice to insert an output transformer between a speaker of low impedance and the output tubes as in *Fig. 1* in order to "match" the speaker to the tubes. The ensuing discussion is intended to be a simple explanation of this matching process and of all the factors that control the frequency response and the distortion introduced by an output transformer.

Loudspeaker voice coils can be wound with sufficient turns to give an impedance of 3Ω–4kΩ and, in fact, these were common in the very early days of radio. However, the difficulties of winding make the cost almost prohibitive, and such a high percentage of the limited space available is occupied with insulation between turns that the efficiency is rather low. It need hardly be stressed that the use of high-resistance wire is an inadmissible solution to the problem for this merely increases the amount of audio power that is dissipated uselessly in heating the loudspeaker voice coil. The efficiency of loudspeakers is already too low for any further loss to be tolerated.

There are other difficulties in the way of connecting the voice coil directly in the anode circuit of a tube or tubes. If used in this way the tube anode current must pass through the voice coil, increasing the amount of power that is dissipated in heating the coil and interacting with the magnetic field in the gap in such a way as to drive the coil out of the gap.

This is a difficulty that is eased but not eliminated by the use of a pair of tubes in push-pull as in *Fig. 2*, for while the anode current can be balanced out in the no-signal condition the balance does not hold at other signal levels, nor does it usually hold for more than a few minutes after the adjustment is made. If a transformer is not used the trouble can only be eliminated by inserting a large blocking capacitor though this would need to be of several thousand microfarads (for a 15Ω loudspeaker) in order to maintain the response at low frequencies.

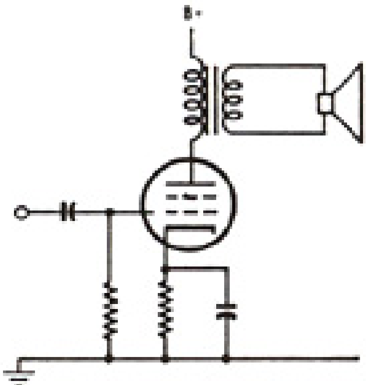

FIGURE 1: Output transformer used to obtain the correct load impedance and isolate the voice-coil current from the plate supply.

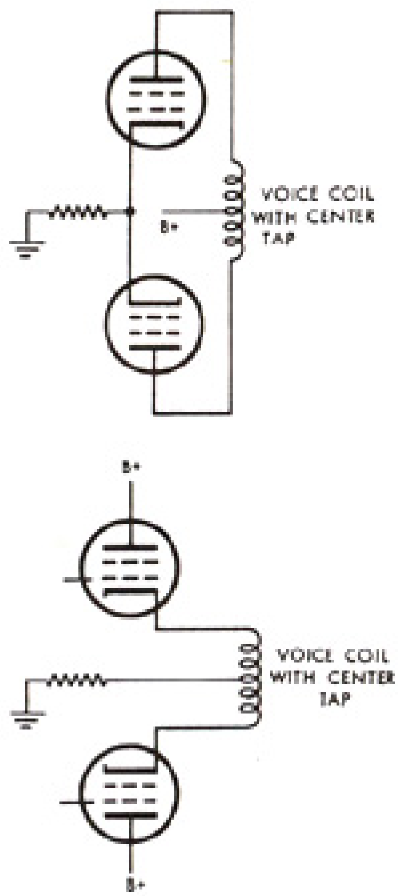

FIGURE 2: Use of push-pull connection balances anode current but does not eliminate difficulties due to current in voice coil.

Both problems—that of removing the anode current from the coil while efficiently raising the voice coil impedance—are solved by the use of an output transformer as in *Fig. 1*. This is the solution that is commonly adopted even though it involves the addition of another relatively expensive component.

Iron cored transformers have a reputation as "distortion introducers" but later in the discussion it will be shown that when properly designed, an output transformer need be no worse than most of the other components in this respect. If the amplifier design is such that the output transformer can be inserted in the feedback loop, the extra distortion introduced is absolutely negligible by any standard.

Functioning of Transformer

First of all let us consider just how an output transformer enables a low-impedance loudspeaker to appear as a high-impedance load in the anode circuit of a tube. The basic transformer merely consists of two coils of wire wound around an iron path common to both coils as in *Fig. 3*. An alternating voltage applied to either pair of terminals results in a current flowing in the coil and the appearance of magnetic flux in the core. At any instant the flux may conventionally be thought of as emerging from the top of the coil, "flowing" around the iron circuit and re-entering the bot-

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ABOUT THE AUTHOR

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tom of the same coil. One half cycle later the direction of flow of both current and flux will be reversed. It is important to note that *all* the flux produced by the current in one coil is guided through the second coil by the iron core.

At this point there emerges a phenomenon that is basic to all transformers; the flux produced by the voltage applied to the input or primary coil will induce in the secondary coil exactly the same *voltage per turn* as exists across the primary coil. Thus if we apply 10V to a primary coil of 100 turns (0.1V per turn) 10V will appear across the terminals of a secondary winding having 100 turns. This is quite independent of the frequency of the applied voltage. If the secondary winding has 1,000 turns a voltage of 100V (0.1V per turn) will appear across its terminals.

However, transformers can neither produce nor store power, they are merely convenient devices for changing voltage levels and therefore (neglecting losses at the moment) all the power put into the primary winding from the power supply must be dissipated in the load resistance connected across the secondary winding. Thus the product of voltage times current ($V_1 \times I_1$) in the primary winding must equal the product

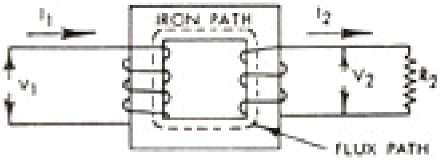

FIGURE 3: Diagram of basic transformer.

of voltage times current ($V_2 \times I_2$) in the secondary winding.

It should now be clear that a transformer with an equal number of turns on both windings (ratio of 1:1) can be inserted between a power source and a load resistance without gaining or losing any power though there are often great practical advantages in isolating two circuits in this manner.

Conditions are apparently slightly different when a transformer having unequal numbers of turns on the two windings is inserted between a power source and a load circuit. The power supplied to the primary side is still equal to the power drawn from the secondary side and thus

$$V_1I_1 = V_2I_2$$

This may also be expressed in terms of the resistance connected across the secondary

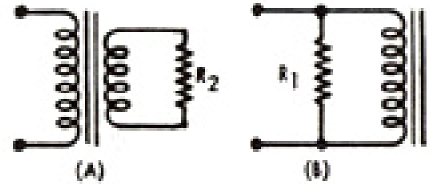

FIGURE 4: When the ratio of primary to secondary turns equals n, the transformer and resistance of (A) can be replaced by (B), where $R_1 = n^2R_2$.

terminals for $V_2I_2 = V_2^2/R_2$. We may also express the power supplied to the primary side in terms of a resistance R_1 which apparently appears across the primary terminals when R_2 is connected across the secondary terminals. Remembering that the power supplied to the primary can only equal the power drawn from the secondary side we have

$$V_1^2/R_1 = V_1I_1 = V_2^2/R_2 = V_2I_2$$

or as we are interested in finding the value of R_1

$$R_1 = \frac{V_1^2}{V_2^2} \times R_2 = \left(\frac{V_1}{V_2}\right)^2 \times R_2$$

However V_1/V_2 is also the turns ratio (primary turns/secondary turns) and if we use the symbol n to denote turns ratio, then

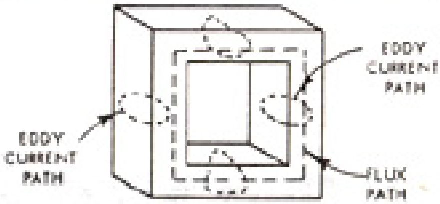

FIGURE 5: Eddy current paths are at right angles to the flux path.

$$R_1 = n^2 R_2 \quad (1)$$

Thus by inserting a transformer between a load resistance R_2 and the tube or any other form of power generator, we can alter the value of the resistor R_2 to n^2R_2. This may be more usefully expressed in another way as we are generally more interested in finding what the transformer ratio n must be in order to make a voice coil of low resistance R_2 look like a much higher resistance R_1. A simple arithmetical fiddle with Equation (1) shows that

$$n = \sqrt{\frac{R_1}{R_2}} \quad (2)$$

and we have the well known formula for determining the turns ratio n of an output transformer when we know the speaker impedance R_2 and the optimum load R_1 for the output stage.

As a typical example of its use, the turns ratio required to match a 15Ω loudspeaker to a pair of EL34 tubes may be worked out. A pair of EL34s requires an optimum anode-to-anode load of 3.4kΩ, and thus the ratio of secondary turns to total primary turns is

$$n = \sqrt{\frac{3400}{15}} = \sqrt{226} = 15{:}1$$

Number of Turns

A second and very important question immediately springs to mind. We know the turns ratio but it is clearly necessary to know the actual number of turns, for a turns ratio of 15:1 would be achieved with 15 turns and 1 turn or 15,000 turns and 1,000 turns. What are the factors affecting the choice? The answer leads on to a whole series of interesting points that should eventually leave us with a very clear idea of all the factors affecting the performance of any output transformer.

So far, a perfect transformer having zero losses has been assumed and therefore (A) and (B) of *Fig. 4* are equivalent in that they both absorb the same amount of power from the tube or other source. This would only be true if the transformer absorbed no power and if all the magnetic flux produced by the primary winding did in fact pass round the iron core and encompass the secondary winding. Both these assumptions are very nearly correct, but it is the small differences between correct and nearly correct that account for all the troubles, though the discrepancies have no effect on the turns ratio problem previously discussed.

The transformer losses can be reduced to a very small fraction of the power handled by the transformer though the average radio receiver uses a transformer that dissipates about 50% of the power supplied to it. Even a loss of 50% might not be very serious if the losses were the same at all frequencies for they could be corrected by merely fitting a larger tube or tubes in the output stage. However, the losses are a function of frequency and the applied voltage, and as the audio signals to be handled by the transformer range over a band from 20Hz–20kHz the frequency-dependent losses can be very important. A diversion to explain the nature of the losses is necessary if we are to understand the question of "how many turns?" but this discussion will be made as painless as possible.

Losses in Transformer

The losses in an output transformer have several components that require individual consideration. They are:

1. The power losses that result from setting up the magnetic field in the iron core. These losses have two components: (a) hysteresis loss, and (b) eddy-current loss.
2. The power loss that results from the current circulating in the copper wire of both primary and secondary windings.
3. At the low-frequency end of the range the transformer diverts some of the available signal current from the load.
4. At the high-frequency end of the range the transformer restricts the current that can be supplied to the loudspeaker by the tubes.

The iron losses will be discussed first of all. An alternating magnetic field in an iron core requires a continuous supply of power to overcome the "magnetic friction" involved in continuously re-orienting the fundamental magnetic particles along the line of the flux path as the polarity of the current flow reverses each half cycle. An indication of the process will be obtained if the iron circuit is considered to be composed of many millions of tiny permanent magnets.

As the current flowing in the primary coil reverses its direction all the individual microscopic permanent magnets reverse their direction and some power must be supplied to overcome their mechanical resistance to this re-orientation. This power loss appears as heat in the iron core in just the same way as power and will be constant at this value at all higher frequencies (though see the later comment about high-frequency response). The requirements for a flat response are now fairly clear; the inductance of L_P should be sufficiently high to ensure that it does not shunt current away from R_L, for when the current in R_L falls the voltage across R_L falls and the frequency response begins to deteriorate.

The more technically minded will see a flaw in this reasoning. At those low frequencies where the reactance of L_P is low compared to R_L, the total circuit impedance will fall and the current drawn from the constant-voltage generator will rise and thus tend to maintain constant the voltage across R_L and L_P.

A detailed analysis shows that this compensating effect can be exactly allowed for by assuming that the generator resistance has a value lower than the slope resistance r_a and in fact is equal to the tube slope resistance r_a and the load resistance R_L in parallel. The equivalent circuit then reduces to that of (E) in *Fig. 6* and has the same voltage/frequency relation for V_O/V_I as the appreciably more complex circuit of (A) in *Fig. 6*, an effective demonstration of the advantages of equivalent circuits.

In a simple circuit such as that of (E) in *Fig. 6*, it is fairly easy to see that V_O will tend to approach V_I as the reactance of L_P becomes large relative to the resistor R_g. The reactance $X_L = 2\pi f L_P$ is directly proportional to frequency and thus it is clearly going to be difficult to maintain X_L large compared to R_g, down to such very low frequencies as a few hertz. When transformers are used there is no alternative to accepting a frequency response that falls away at low frequency, but the frequency at which the falloff commences can be moved down to any desired frequency by increasing the value of L_P.

Shapely Responses

For reasons that will emerge later, it is usual to take as the cutoff frequency that frequency at which the reactance of L_P equals the resistance R_g, this being the frequency at which the output is down by 3dB. This is an arithmetical simplification rather than the point in the frequency range at which there is a significant cutoff, for the power output is only falling away at the rate of 6dB/octave.

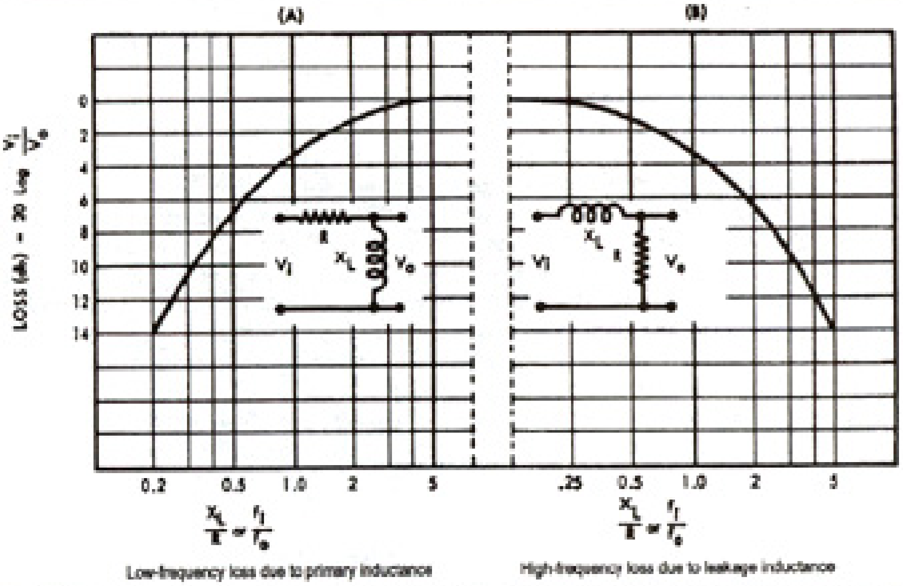

FIGURE 7: Basic output transformer characteristics. f_O is the frequency at which the reactance X_L is equal to the resistance R, and f_1 is the frequency at which the loss is to be determined.

The *shape* of the frequency response, i.e., the relation between the ratio V_O/V_I and frequency, is controlled by the ratio of X_L to R_g and thus is unalterable. All output transformers have the same shape of frequency response, but a good transformer is up to its level value at a very low frequency whereas a poor transformer does not achieve its "flat" value until a much higher frequency is reached. It is convenient to display this universal response in the form of a single curve (*Fig. 7*), f_O being the frequency at which the reactance X_L of L_P equals the resistance R_g. From this it will be seen that at this cutoff frequency where $f_1/f_O = 1$, the loss is 3dB, but at half this frequency the loss is only 7dB.

Some realism is put into the picture by taking a look at the sort of values of primary inductance L_P that are required in practice if a flat frequency response is to be obtained. The two EL34s used in the earlier example in Part I require an anode-to-anode load of 3.4kΩ and have a quoted slope resistance r_a of 15kΩ, though as a push-pull stage is being considered the effective source resistance can be taken as 30kΩ.

This is some ten times the required anode-to-anode load, a relation typical of tetrodes and pentodes and it results in the effective generator resistance R_g being 3kΩ, only slightly lower than the required anode-to-anode load 3.4kΩ. If it is decided to allow a loss of 3dB at 50Hz the reactance of the primary inductance L_P must also be 3kΩ at this frequency and L_P is then 3,000 $(2\pi \times 50)$ = 10H approximately. If the -3dB point must be at 10Hz, then the inductance must be five times higher, or 50H.

Practical Design

While dealing with a specific example it is worth calculating the number of turns and the size of transformer required to obtain the primary inductance suggested. Any required value of inductance can be obtained in either of two ways; a small number of turns on a large iron core, or a large number of turns on a small iron core. A large core and few turns should result in small signal power losses and

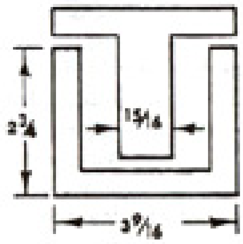

FIGURE 8: Typical lamination shape.

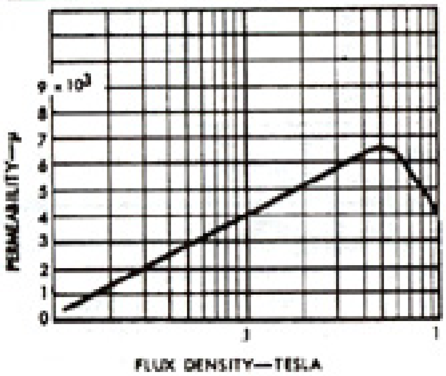

FIGURE 9: Typical relation between core flux density and permeability for transformer steel.

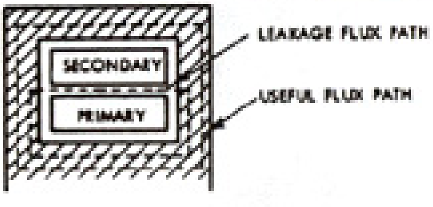

FIGURE 10: Secondary wound over primary. Suitable only for transformers dealing with a restricted frequency range.

lost in mechanical friction appears as heat. A more detailed study shows that these hysteresis losses are directly proportional to the signal frequency and to the (flux density)$^{1.6}$ [*Superscripted numbers refer to proportional values throughout.—Ed.*] The frequency is that of the audio signal and cannot be altered but the hysteresis losses can be minimized by working the transformer core at a low magnetic flux density.

Eddy-current losses are the second component of the iron loss and are found to be proportional to (flux density)^2 and (frequency)^2. Their origin is interesting. Very early in the present discussion it was noted that an emf (electromotive force) was induced in the turns of a coil by the alternating magnetic field "flowing" through the coil. On looking again at *Fig 5* it will be seen that the iron core itself constitutes a large single turn and thus current will tend to flow in a circular path across the iron core section at right angles to the direction of the magnetic field.

Power is absorbed from the source to supply the I^2R losses due to this current flowing in the iron path. The circulating current and hence the losses can be reduced by increasing the resistance of the path taken by the current, a result that is generally achieved by laminating the iron circuit so that the circulating current must pass across the relatively high resistance contact between laminations. It was also noted that the losses are proportional to (flux density)^2 and thus the losses can be greatly minimized by designing the transformer to work with a low flux density.

The copper losses need little description. If a current of I amps flows in a resistance of R ohms, then there is a total power loss of I^2R watts dissipated as heat in the resistance. In an audio transformer there are power losses in the primary coil due to the signal current and due to the current required to supply the iron losses. In the secondary winding there are losses due to the signal current flowing around the voice-coil circuit. A more complete analysis shows that all the losses are not so simply explained as in this preliminary discussion but the explanation is adequate at this stage.

The losses grouped together under Item 3 are perhaps a little more troublesome to understand but the problem is greatly eased by the introduction of an equivalent circuit, a technique much used in the study of tube circuits. The basic practical circuit of an output transformer in the anode of a single tube is that of *Fig. 1*. The first step in producing an equivalent circuit is to remove all those items that do not affect the signal-frequency performance of the circuit, the aim being to simplify the circuit by reducing it to the bare essentials in order that the effect of each component should be more clearly seen and understood. ✥

OUTPUT TRANSFORMERS
PART II*

BY JAMES MOIR

[Readers should refer to GA 2/94, p. 24, for the first part of James Moir's definitive look at the performance characteristics of output transformers—Ed.]

Though it seems a drastic step, all the high-voltage circuitry can be removed. Indeed when the performance of the tube and transformer is being considered, the whole of the tube circuit—tube, bias resistor, and its shunt capacitor, grid capacitor, and grid resistor—can be removed and replaced by a single resistor having a value equal to the slope resistance r_a of the tube under its working condition.

However, the tube is an active device in that it produces signal power and thus we have to add to our slope resistance r_a a generator that we can assume to produce the same power as the tube. When this is done the whole of the circuit inside the dotted box can be replaced by the two devices in (B) of *Fig. 6*, a resistor r_a and a generator, the combination appearing as a power generator having no resistance in series with a resistance equal to the tube slope resistance.

A Leap of Faith

The output transformer itself is again a little more troublesome. The practical circuit is that of (A) in *Fig. 4*, two separate windings coupled by the iron core with the second winding supplying power to the loudspeaker. A start can be made by substituting a resistor R_{LS} for the voice coil to give (B) of *Fig. 4*, but at the moment the next step will have to be taken on trust for later verification. The transformer ratio, usually denoted by the symbol n, has no effect on the frequency response, so to avoid having to multiply every impedance by n^2 it is simpler to assume that the turns ratio is 1:1, the two windings having equal numbers of turns.

As was seen in Part I, a load resistor of, say, 1kΩ connected across the secondary of a 1:1 transformer has exactly the same effect as the same resistance connected across the primary winding, at least at the low-frequency end of the audio range. The practical circuit has now been reduced to the much simpler circuit of (C) in *Fig. 6*, the transformer and speaker voice coil being reduced to a resistor R_L and an inductance L_P in parallel, the inductance being that of the transformer primary winding measured at some low audio frequency with the secondary winding open-circuited.

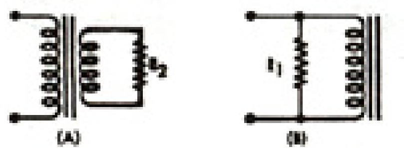

FIGURE 4: When the ratio of primary to secondary turns equals n, the transformer and resistance of (A) can be replaced by (B), where $R_1 = n^2R^2$.

If the generator is assumed to produce constant volts at all audio frequencies, the variation of voltage across R_L and L_P will follow exactly the same law as the variation with frequency of the voltage across the loudspeaker voice coil in the practical circuit. This is the simplification that is desired.

A Simpler Circuit

Even without putting values on R_L, it is easy to see the sort of frequency response that will be obtained at low frequencies and to get an idea of the design steps that are necessary to get a flat response. With the generator (an AF oscillator) set to a near-zero frequency, current will flow around the circuit and the generator voltage will be dissipated across r_a in series with R_L and L_P in parallel. The voltage across R_L and L_P will only be a small fraction of the total generator voltage for the reactance of L_P ($X_L = 2\pi f L_P$) will be small.

As the generator frequency is increased, the reactance of L_P will increase (being directly proportional to frequency) until at some higher frequency the reactance will be much higher than R_L. At, and above this frequency, the inductance L_P can be removed for it has no effect on circuit performance and the circuit then consists of the generator and two resistors, and R_L.

At these frequencies the equivalent circuit r_a will be that of (D) in *Fig. 6*, the output voltage will clearly be

$$V_o = V_i \times \frac{R_L}{r_a + R_L}$$

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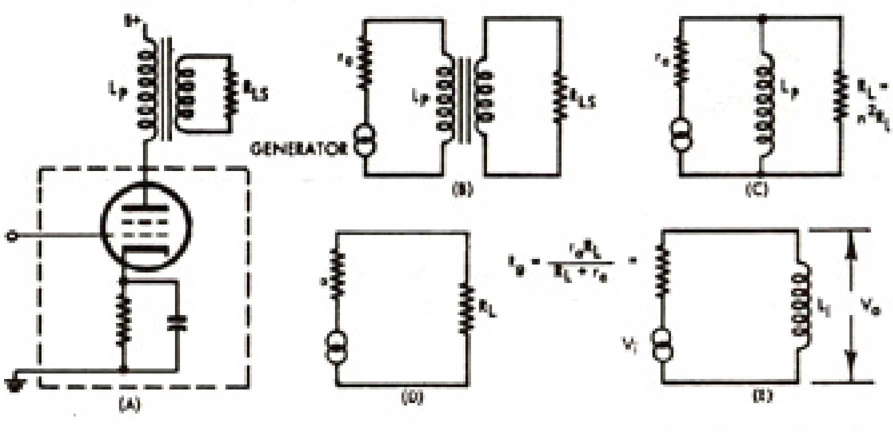

FIGURE 6: Practical single-ended output circuit (A), and its equivalents; (B), simplest form; (C), at low frequencies; (D), between 150Hz and 4kHz; (E), final equivalent circuit of frequencies below 150Hz.

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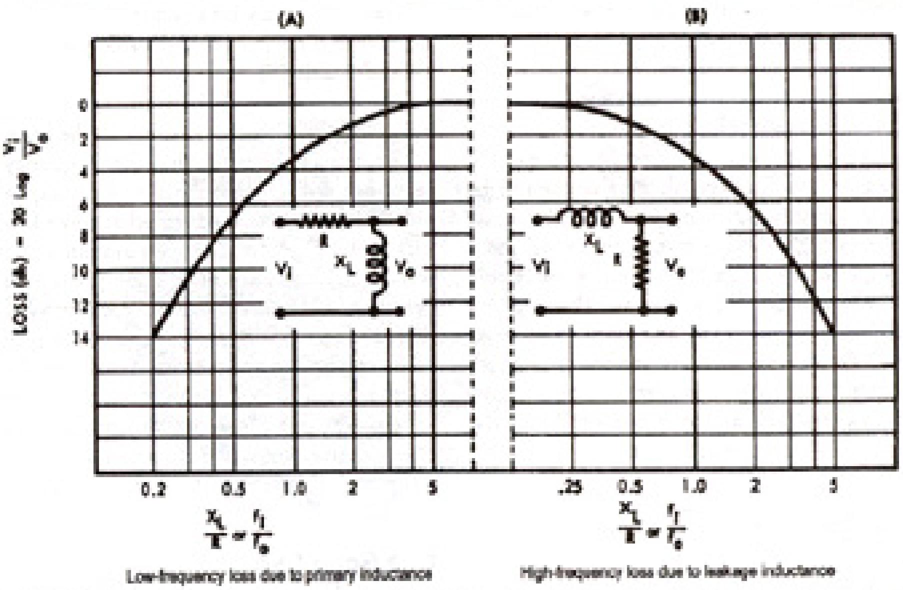

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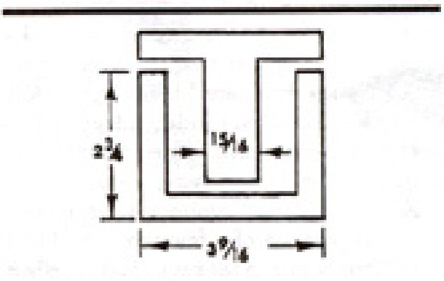

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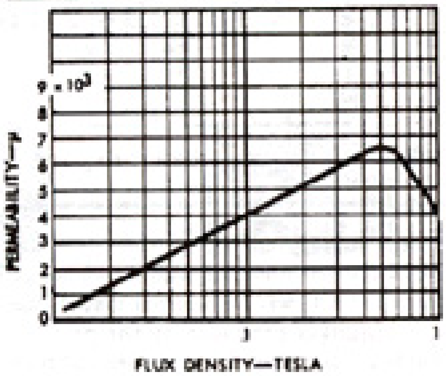

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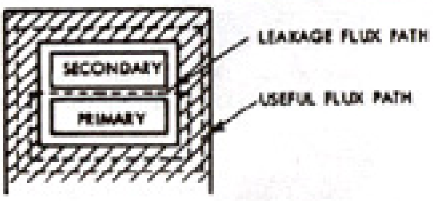

FIGURE 10: Secondary wound over primary. Suitable only for transformers dealing with a restricted frequency range.

a high price, with the converse being true if a small core and a large number of turns are adopted. The choice of core size is thus somewhat arbitrary unless the permissible power loss can be specified.

An examination of the lists of some of the leading manufacturers shows that their high-quality transformers have an overall volume of about $2in^3/W$, suggesting that a 1½" stack of the laminations shown in *Fig. 8* will handle 20W, though this is a point that will be checked more closely at a later stage when the distortion products are being studied. The inductance of a coil wound on a closed iron core (such as *Fig. 3* in *Part I*) is given (but only approximately) by

$$L = \frac{3.2T^2 \mu A}{10^8 \times l} \qquad (3)$$

where T = number of turns
A = cross-sectional area of core
μ = permeability of core
l = length of flux path
(all dimensions in inches).

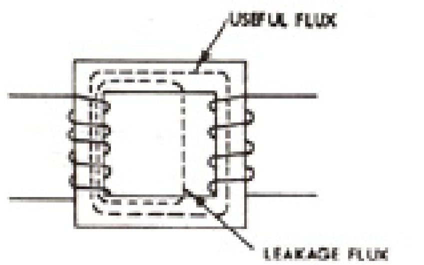

FIGURE 11: Paths of working and leakage fluxes in basic transformers.

All the factors that appear in this formula are unambiguous except μ, the permeability of the core material, and this is difficult to specify because the permeability of any of the usual core materials is a function of the core flux density. Typical variations in permeability with flux density taken from the data sheets in a manufacturer's lists are illustrated by *Fig. 9*, but experience shows that these permeability values are not achieved under working conditions. Data for these curves are invariably taken on ring samples without air gaps and after annealing.

Laminations punched from the same material are rarely annealed after punching, are then assembled with air gaps that are small but unavoidable and finally used in transformers that carry small unbalanced anode currents, all important factors in reducing the permeability below the ring-sample value. It is more realistic to use permeability values that are half those read from *Fig. 9* when calculating the winding inductance from equation (3). The permeability will be seen to vary by a factor of about five times over a range of flux densities between .02 and .5 tesla (T).

The inductance of the primary winding will also vary with flux density by the same factor of five times, so that the frequency response will change with power output unless the inductance measured at some low flux density is adequate to maintain a flat response. The choice of an appropriate flux density and the related value of permeability is somewhat arbitrary, but if a value for μ of 1,500 corresponding to a core density of .5T is used, the final performance is likely to be very acceptable. However, this problem of core density will come up again at a later stage when harmonic distortion is being considered.

One Good Turn ...

Using the $2in^3/W$ figure it might be expected that a 1½-inch-deep stack of the laminations shown in *Fig. 8* would handle 20W with ease. The core area of a 1½" stack is roughly $1½in^2$ and the iron path length 8". Inserting these values into equation (3) shows that about 1,100 turns are required to give an inductance of 10H while 2,400 turns are necessary to obtain 50H. When harmonic distortion is considered at a later stage, it will be shown that in general the primary inductance required to hold harmonic distortion to an acceptably low limit automatically ensures a good frequency response.

The specified number of turns can be wound on to the core as a single coil having the secondary turns wound on top as in *Fig. 10*, though this is not the usual practice when a transformer having a high-quality performance is required. Why is this simple (and therefore low priced) construction not adopted? The answer is that the relative disposition of the two windings on the core controls the high-frequency performance, an aspect of the design problem that can now be considered.

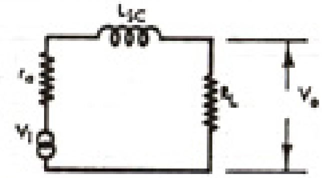

FIGURE 12: Equivalent circuit at frequencies above 4kHz.

It is best approached by referring to the Part I discussion dealing with the choice of turns ratio. It was then stated that all the magnetic flux produced by the primary winding was confined to the core and thus interlinked both coils. When the turns ratio and number of turns are being considered, this assumption is perfectly valid, but when the high frequency performance is under examination the assumption is too sweeping.

In the simple example of *Fig. 11* magnetic flux lines emerging from the top of the coil have two alternative paths that can be followed back to the bottom of the coil. The designed path is that through the iron core, the path that is followed by the great majority of the magnetic flux. However, a very small proportion of the total flux leaks out of the iron and follows paths through the air as indicated, with the result that all the flux from the primary winding does not link with all the turns from the secondary winding.

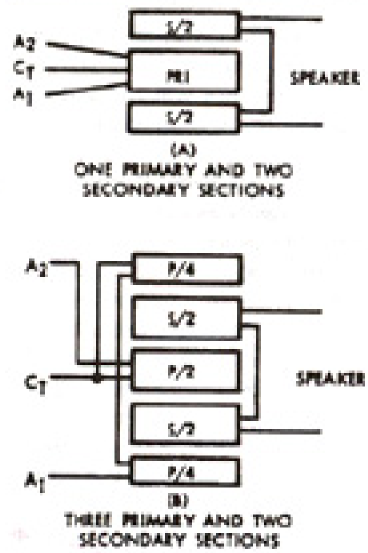

FIGURE 13: Further subdivision of windings to reduce leakage reactance.

In a good transformer as much as 99.9% of the flux from the primary winding links with the secondary, but the remaining 0.1% is responsible for the majority of the high frequency losses. A return to the equivalent circuit of (C) in *Fig. 6* will ease the explanation.

A Fog Lifts

The primary inductance L_P appears in parallel with the load resistance R_L, but above a quite low frequency (50–150Hz) the reactance of this inductance becomes so high in comparison to the resistance R_L that the current shunted off the load resistance becomes quite negligible. Above this frequency, L_P has no effect on the frequency response, which is then determined by the resistances r_a and R_L and is thus independent of frequency, the conclusion arrived at when discussing the low-frequency performance.

A flat frequency response is maintained up to frequencies of a few thousand hertz but it then begins to fall away again, an effect that is not predicted by the equivalent circuits as they stand in *Fig. 6*. The missing element is an inductance that represents the effect of the magnetic flux which strays from the iron path and thus fails to link both coils. It is omitted from *Fig. 6* because it has no effect on the performance of the transformer at low frequencies.

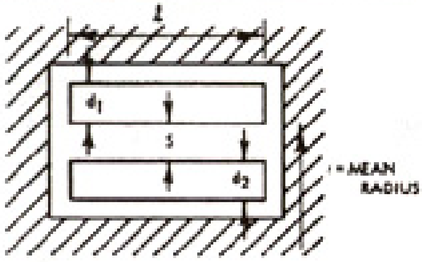

FIGURE 14: Dimensions required for calculation of leakage inductance.

The clearest mental picture is obtained by assuming that the whole of the flux produced by the primary winding bypasses a few of the secondary turns, leaving these few turns as an inductance outside the transformer and in series with the secondary load resistance R_L. It is really immaterial whether we consider that 99% of the flux links with 100% of the secondary turns or that 100% of the primary flux links with 99% of the secondary turns, for it is the product of (flux) × (turns) that is important; but a clearer picture of the process is given by the second approach.

The inductance that exists as a result of the failure of the primary flux to link all the secondary turns is generally known as the leakage inductance and can be measured on any of the standard AC bridges by short circuiting the secondary terminals and measuring the inductance that appears at the primary terminals. The same final answer is obtained if the primary terminals are shorted and measurements made at the secondary terminals, but the two measurements will differ in the ratio of the (turns ratio)^2. [*Superscripted numbers refer to proportional values throughout.—Ed.*]

The general effect of this leakage inductance on the frequency response is now fairly easily seen from a consideration of its position in the equivalent circuit where it appears in series with the secondary load resistance R_L as in *Fig. 12*. As the signal frequency rises, the reactance of L_{SC} rises proportional to frequency, eventually becoming comparable in value to the secondary load resistor R_L and with further increase in frequency the reactance of L_{SC} will exceed R_L. The signal voltage produced by the generator is now divided between three circuit elements—r_a the equivalent resistance of the generator, L_{SC} the leakage inductance, and R_L the secondary load resistance--and therefore V_O will fall off with increase in frequency at the rate of 6dB/octave.

The point in the frequency range at which the falloff begins to be significant (rather arbitrarily, the frequency at which the response is 3dB down) is a function of the ratio of the reactance of L_{SC} to the combined total circuit resistance $r_a + R_L$. When $X_{SC} = 2\pi f L_{SC} = r_a + R_L$ the loss is 3dB and increases at the rate of 6dB/octave as shown at (B) in *Fig. 7*. The similarity between the relations governing the high-frequency loss and those governing the low-frequency loss will be apparent on comparing (A) and (B) in *Fig. 7*.

Leakage Reactance

Clearly if the response is to be well maintained up to the highest frequencies, L_{SC} must be reduced to a minimum so the factors that affect L_{SC} will now be considered. Little thought will be required to decide that the leakage inductance will increase as the number of turns on the windings increase, following the normal law that inductance is proportional to (turns)^2.

Advantage cannot be taken of this relation to reduce the leakage inductance, for as we have seen earlier the total turns are fixed by the response that is desired at the low-frequency end of the range. The alternative course of action is to reduce the amount of leakage flux from the primary that fails to couple with the secondary winding. This is a question of bringing the secondary winding as close to the primary winding as physically possible. Some possibilities will be considered.

The worst possible arrangement is that of the elementary transformer of *Fig. 11*, where the primary winding is arranged on one limb and the secondary winding on the other limb. Leakage flux then follows the path shown and may amount to an appreciable fraction of the total flux. It may be greatly reduced by winding the secondary on top of the primary winding as shown in *Fig. 10* and abandoning the core type of lamination shown in *Fig. 11* in favor of the shell type of *Fig. 8*. Magnetic leakage then follows the path shown in *Fig. 10* and will obviously be a great deal less than in the simple arrangement of *Fig. 11*.

Further reduction in leakage may be achieved by dividing either winding into two halves and disposing them about the other winding. This technique of subdivision may be carried still further, both primary and secondary windings being subdivided into sections and interleaved. Some typical arrangements are shown in *Fig. 13*. That of (B) has particular advantage in push-pull circuits in that the two half primaries can be made to have the same resistance by using P/4 and P/4 in series for one half primary, with P/2 for the other half. This also equalizes the leakage inductance from either half primary into the secondary.

Two alternative approaches to the problem of reducing leakage inductance are worthy of comment. Reduction of the spacing between the primary and secondary sections is clearly an advantage, but a limit to this technique is set by the necessity of providing interwinding insulation between the sections capable of withstanding the plate supply voltage and signal voltage excursions. It is usual to operate amplifiers with the secondary winding at or very near ground potential but with the primary winding at B+ potential. The newer insulations with high dielectric strengths offer considerable advantages in reducing the thickness of the intersection insulation.

The leakage inductance of any particular arrangement of coils can be calculated with a moderate degree of accuracy and it is worthwhile examining the relationship for the light it throws on the factors responsible for leakage. A simple formula that gives good agreement with measured values is

$$L_{SC} = 3 \cdot 2 \times T^2 \times \frac{2\pi r}{l}\left(s + \frac{d_1}{3} + \frac{d_2}{3}\right) \times 10^{-3}\,\mathrm{H}$$

the symbols having the meaning shown in *Fig. 14*.

From this it will be seen that the leakage inductance is increased by an increase in the radius r of the winding; by an increase in S, the spacing between coils; or decreased by an increase in *l*, the wound length of the coil.

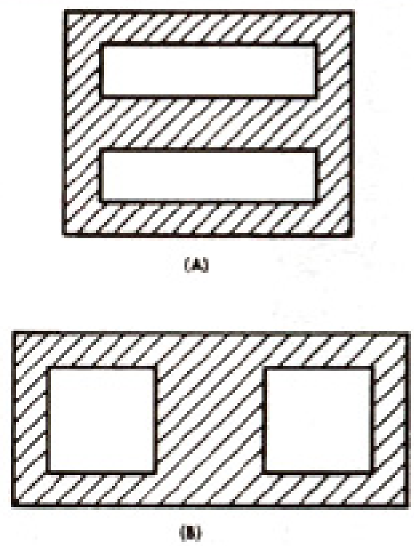

FIGURE 15: Laminations having long windows (A) have lower leakage inductance than those having square windows as at (B).

A lamination having a long narrow window such as that of (A) in *Fig. 15* will give a lower leakage inductance per turn than one with a square window such as that at (B). This is not quite the advantage that it appears at first sight, for laminations with long windows tend to have long iron paths and thus have a lower primary inductance L_p per turn than one with a square window. Nevertheless, there is an advantage to be gained by an appropriate choice of lamination shape.

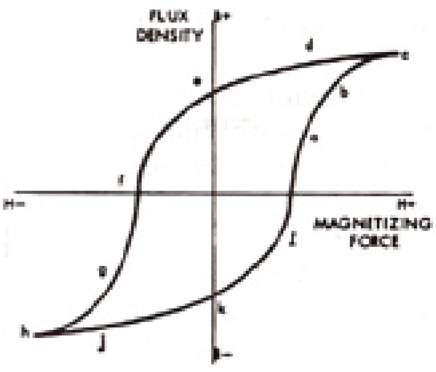

FIGURE 16: Typical B/H relation for iron laminations.

Distortion

The last performance characteristic to be discussed is the generation of harmonics and intermodulation distortion by an iron-core device. This is not such a well understood subject and in consequence will be covered in rather greater detail than was thought necessary for some of the earlier characteristics.

How does distortion arise in an iron-cored device? Fundamentally it is due to the nonlinear relation between the magnetizing force H and the resultant flux density B produced in the iron core, but it is also due to the presence of hysteresis in magnetic materials.

In an ideal magnetic material, the magnetizing force H would produce a magnetic flux density B proportional to H. Thus if H were doubled (by doubling the current or the number of turns) B should double. Moreover, B should have the same value for any particular value of H, irrespective of the direction in which the current flows in the magnetizing coil. Neither of these requirements is met in a magnetic circuit that consists wholly of magnetic material. What does happen is illustrated by *Fig. 16*, a typical B/H relation for a transformer steel.

Starting from zero current in the magnetizing winding, but with the iron path magnetized by the previous cycles of the supply, the flux density in the iron rises roughly proportionately to the current up to point *a* in *Fig. 16* then less than proportionately from *a* to *b*, and finally saturates at *c*; very large increases in magnetizing current are then required to produce very small increases in flux.

If the direction of the current flow is reversed, B commences to fall, not along the path *c,b,a*, but along a new path *d,e,f*, where the values of B are always higher than were obtained for the same values of H when H was increasing. The magnetizing current must be *reversed* to reduce B to zero at *f*, a symmetrical path *g,h,i,k,l* then being traced as H increases to a negative maximum, reverses, and returns to zero.

Cul-de-Sac

The important point is that the path followed by the value of B encloses an area instead of merely following a straight line. From the B/H relation of *Fig. 16* it may be deduced that a sinusoidal magnetizing current in the primary coil will not produce a sinusoidal flux waveform in the iron circuit. As the secondary voltage is proportional to the rate of change of flux, a sinusoidal secondary voltage can only be produced by a sinusoidal flux waveform and this will in turn only be produced by a nonsinusoidal current wave in the primary winding.

At this stage it would appear that an impasse has been reached, for distortionless reproduction demands that a sinusoidal voltage on the grid of the output tube produces a sinusoidal voltage across the output transformer secondary, though it has been shown that this result can only be achieved by supplying a nonsinusoidal current to the transformer primary. However, the impasse is only the result of shallow thinking about the problem as it may be shown that if the impedance of the source is very small, a sinusoidal voltage applied to the transformer primary will result in a nonsinusoidal primary current, a sinusoidal flux waveform, and a sinusoidal secondary voltage. The significant point is contained in the phrase "if the resistance of the source is very small" and the question that immediately jumps to mind is, how small?

There are few abrupt discontinuities in nature and it is unlikely that the distortion will prove to be zero when the source resistance is zero and yet rise sharply for very low values of source resistance. Common sense is right on this point.

A detailed analysis shows that the percentage distortion is related to the ratio of circuit resistance to the inductive reactance of the primary winding of the transformer and is a function of the maximum flux density at which the iron is worked. This latter result is to be expected for there is a fair degree of proportionality between H and the resultant B if the flux density is not allowed to exceed point *a* in *Fig. 16*. Unfortunately a low flux density means a large core and an expensive transformer.

Handling Harmonic Distortion

The advantages of a low-resistance source in minimizing harmonic distortion are less obvious and need a more detailed explanation. If a generator of zero resistance and a sinusoidal voltage waveform supplies current to a resistive load, both the current and voltage have sinusoidal waveforms. When the resistance load is replaced by an inductance the voltage waveform remains sinusoidal but the current waveform is distorted by just the right amount to produce a sinusoidal flux waveform and thus a sinusoidal secondary voltage waveform. The primary current wave

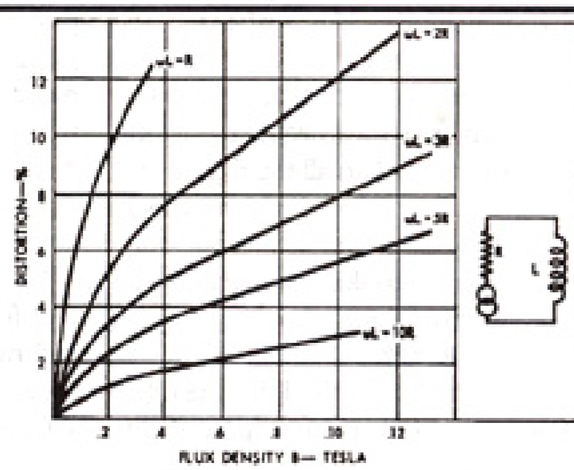

FIGURE 17: Third-harmonic distortion in the voltage across L as a function of the flux density B for values of ω_L/R.

Appendix

The flux density B in a transformer core can be calculated from the following equation

$$B=\frac{V\times 10^{8}}{4.44\,fTA}\text{lines / cm}^2$$

where B = flux density
V = voltage across winding
f = frequency
T = number of turns on winding
A = core area cm^2

In the example used in this article V is the voltage developed across the anode load R_L of 3.4kΩ at the rated output power of 20W. This is

$$V=\sqrt{WR_L}=\sqrt{20\times 3400}=260V$$

Using a core having an area of 1.5in² ($10cm^2$) and the 2,400-turn winding, the core flux density B at frequency 50Hz is

$$B=\frac{260\times 10^{8}}{4.44\times 50\times 2400\times 10}=4900\text{ gauss, or .49T}$$

At this value of flux density the effective permeability may be taken as 3,200 and the inductance of the 2,400-turn winding is then

$$L_p=\frac{3.2\times 2{,}400^2\times 3{,}200\times 1.5}{8\times 10^{8}}=110H$$

L at 50Hz = 34,400 making ωL/R = 11.5.

The distortion where B = .49 and L/R = 11.5 is, from *Fig. 17*, approximately 1.6%.

is then found to contain a high percentage of third, fifth, and seventh harmonics.

In an intermediate condition when the circuit contains some resistance, the current drawn from the source is less distorted but the distortion of the voltage waveform is increased. In general any resistance in the circuit prevents the inductance drawing the harmonic currents it requires to maintain a sinusoidal flux waveform. If a sinusoidal flux waveform is not maintained then the waveform of the secondary voltage will be nonsinusoidal.

When considering the low-frequency response of a transformer it was shown that the resistance that controls the response is the parallel combination of the source and load resistances. The same parallel combination also controls the harmonic distortion that is generated. If the transformer is a poor example with high resistance windings these winding resistances must be added to the load resistor before working out the parallel combination.

Most of the manufacturers of transformer steels have produced curves showing the relationship between distortion and the ratio of the effective circuit resistance to the reactance of the primary winding. Typical curves for a 4% silicon steel commonly used in high quality transformers are shown in *Fig. 17*. The most significant information to be obtained from the curves is the very high distortions that occur even at low flux densities when the source resistance is comparable with the reactance of the transformer primary winding. Earlier in this article I showed that a transformer having a primary inductance of only 10H would have a frequency response only 3dB down at 50Hz when used with two EL34 tubes working into a load of 3.4kΩ. It is interesting to calculate the harmonic distortion that is produced if such a transformer is used.

At a frequency of 50Hz an inductance of 10H has a reactance of 3.14kΩ, approximately equal to the effective generator resistance when using two EL34s in push-pull. The left hand curve for ωL = R is then appropriate. The core flux density when the power output is up to the rated figure of 20W may be computed (see *Appendix*) to be approximately 1T, a value that is well off the curve but it will be seen that the harmonic distortion is up to 12% for a flux density of only .3T and continues to increase rapidly at higher flux densities, a quite intolerable result for a high-quality transformer.

The alternative discussed was to use a transformer having a primary inductance of 50H and thus having a response that is flat down to about 10Hz. Earlier in this article we determined that such an inductance would be achieved with a primary winding of 2,400 turns on the same core. The increased turns bring the core flux density on full load (20W) down to about .45T, the core material having an effective permeability of about 3,200 at this flux density. The resultant primary inductance has then risen to about 110H, making the ratio of primary reactance to effective source resistance approximately 11.5 at 50Hz. Extrapolating the curves on *Fig. 17*, we find that the harmonic distortion is about 1.7% at full load, a very considerable improvement in performance.

Winding Down

These figures make it quite clear that when a high-quality amplifier is being designed, the frequency response must extend well below the nominal lower frequency limit required by the signal spectrum if harmonic distortion is not to be intolerably high on low-frequency signals. In this particular, though typical, example, the response must be flat down to 10Hz in order to achieve distortion values as low as 2% at 50Hz.

The reduction of flux density appears advantageous in reducing harmonic distortion but to a great extent this is an illusory advantage. Provided that the flux density is kept below about .5T at full rated power, little is to be gained by further reduction, for though reference to *Fig. 17* would suggest that the distortion is falling as the flux density is reduced, it must be remembered that μ and in consequence the primary inductance L_p and the ratio of ωL to R is also falling. Thus there is no very significant reduction in harmonic distortion percentage as the maximum flux density is reduced. None of the alternative core materials at present available offer hope of any significant improvement in this situation.

The *Fig. 17* curves also suggest that distortion can be greatly reduced by decreasing the effective resistance of the source. At first thought, tetrodes and pentodes appear appreciably worse than triodes in this respect but further investigation does not always support this view. Two EL34s have an effective slope resistance of 30kΩ as pentodes in push-pull but only 6kΩ connected as a pair of triodes, but it should be remembered that the effective source resistance from the point of view of harmonic generation is the parallel combination of tube resistance and load resistance.

As pentodes, two EL34s require an anode to anode load of 3.4kΩ, making the effective source resistance about 3kΩ. As triodes the tube slope resistance had dropped to 6kΩ but the optimum load has risen to 10kΩ, making the effective source resistance about 3.7kΩ. Thus in this instance triode-connected tubes are slightly worse than the same tubes pentode-connected.

Ultralinear operation of pentode or tetrode tubes offers a significant reduction in effective source resistance, another reason for the obsolescence of "straight" operation of pentodes or tetrodes. Negative feedback, either over the whole amplifier as a distortion reducer, or from the anodes of the output tubes back into the cathode circuit of an earlier tube as a source impedance reducer, has great advantages and is in fact the only method of obtaining distortion values in the region of 0.1% at full rated load. ✧