The Basic Design of Constant Resistance Crossovers

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An analysis of the response and phase characteristics of constant-resistance crossover networks worked out for filters employing from one to four elements.

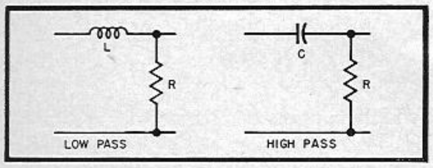

Fig. 1. Basic single-element filters for parallel combination at input into composite filter.

Although crossover filters are now in everyday use, confusion about their design and performance still persists. Most presentations published to date concentrate attention on the filter-derived types, and if the constant-resistance variety is mentioned at all, it is just thrown in for good weight. In point of fact, the constant-resistance type of response offers several advantages over the filter-derived types.

When the parameters are correct for constant resistance, other useful properties appear that are not available with other derivations. The principal properties of these networks may be summarized as follows:

Attenuation Response

(*i*) The responses of the two filters, considered separately, are complementary, so that the sum of the energies delivered to the two output circuits, for constant-voltage or constant-current input (according to configuration), is constant at all frequencies.

(*ii*) Also the attenuation response of each filter gives the maximum flatness within its pass range, with the most rapid transition possible, with the number of elements used, short of introducing peaking in the vicinity of crossover.

Phase Response

(*iii*) The phase response of each filter is symmetrical about cut-off frequency (which is crossover for the combination).

(*iv*) The difference in phase between the signal delivered to the two outputs is constant.

Input Impedance

(*v*) This, for the combined filters, is constant and resistive throughout the spectrum, provided, of course, that each filter is correctly terminated at its output. It is from this property of the filters that they derive their name.

This article will derive the formulas for constant-resistance filters using up to four elements in each filter, showing how the foregoing qualities apply for each case, and will consider the properties of simpler or more complex filters from the viewpoint of their effect on the fidelity of reproduction. At this stage, filters using physical inductances and capacitances will be assumed, but the theory developed will be useful for application to circuits simulating the responses by other means, to be considered later.

For developing the formulas the filters will always be of such a configuration that their inputs are connected in parallel. The results are quite simply applied to filters of the type used for series combination by the principle of duality.

Single-Element Filters

Consider first the single reactance filters of *Fig.* 1, taking first the low-pass section. The relation between input and output voltage is

$$A_L = 1 + j\frac{\omega L}{R}$$

which can also be written

$$A_L = 1 + jx \qquad (1)$$

on the assumption that $\frac{\omega L}{R} = x$.

The energy supplied to the low-frequency unit is V^2/R, and V is E/A_L. So the energy supplied can be written

$$P_L = \frac{E^2}{R} \cdot \frac{1}{1+x^2} \qquad (2)$$

The attenuation response is given by

$$db_L = 10 \log_{10}(1+x^2) \qquad (3)$$

The phase relation between input and output is given by the angle whose tangent is the ratio between the real and imaginary parts of *(1)*. Using the convention that positive angles represent delay, the phase-transfer characteristic is given by

$$\phi_L = \tan^{-1} x \qquad (4)$$

The input impedance of the low-pass unit is

$$Z_L = R(1+jx) \qquad (5)$$

or, since it is to be connected *in parallel with the other unit,* the admittance is required,

$$Y_L = \frac{1}{R} \cdot \frac{1}{1+jx} = \frac{1}{R} \cdot \frac{1-jx}{1+x^2} \qquad (6)$$

Turning now to the high-pass section. The relation between input and output voltage is

$$A_H = 1 + \frac{1}{j\omega CR}$$

which can be written

$$A_H = 1 - j\frac{1}{x} \qquad (7)$$

on the assumption that $\frac{1}{\omega CR} = \frac{1}{x}$.

The energy supplied to the high-fre-

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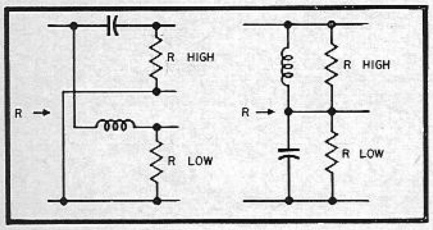

Fig 2. The two forms of composite filter, giving constant-resistance characteristics with single-element sectons. Each reactance is equal to R at crossover.

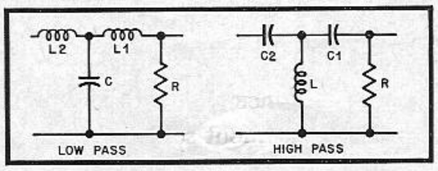

Fig. 5. Basic three-element filters.

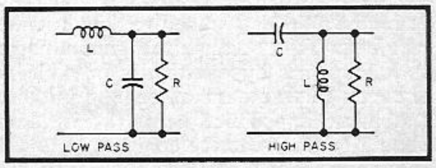

Fig. 3. Basic two-element filters for parallel combination at input to form composite filter.

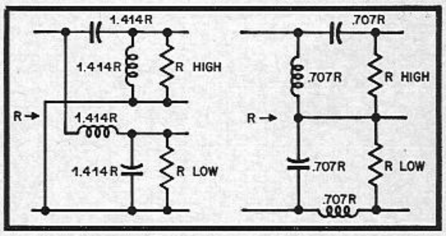

Fig. 4. The two forms of composite filter giving constant-resistance characteristics with two-element sections. Reactances at crossover are indicated.

quency unit can be written

$$P_H = \frac{E^2}{R} \cdot \frac{x^2}{1+x^2} \qquad (8)$$

and the attenuation response is given by

$$db_H = 10 \log_{10} \left[\frac{1+x^2}{x^2}\right] \qquad (9)$$

Adding together the energy supplied to the two units when the filters are paralleled, as given by (2) and (8),

$$P = P_L + P_H = \frac{E^2}{R} \cdot \frac{1+x^2}{1+x^2} = \frac{E^2}{R} \qquad (10)$$

This proves that condition *(i)* applies, i.e. the sum of the energies is constant. It might also be assumed to demonstrate condition *(v)*, because the denominator is R, but it is hypothetically possible to transfer this amount of energy without the input impedance being R, if there should be any reactive component to the input impedance, so this will be investigated.

The phase of the high-frequency output compared to the input is

$$\phi_H = \tan^{-1} - \frac{1}{x}$$

which can be transposed to

$$\phi_H = (\tan^{-1} x) - \frac{\pi}{2} \qquad (11)$$

It is seen that the difference between *(4)* and *(11)* is a constant angle of $\pi/2$ or 90 deg.

The impedance of the input to the high-pass unit is

$$Z_H = R\left(1 - j\frac{1}{x}\right) \qquad (12)$$

whence the admittance is

$$Y_H = \frac{1}{R} \cdot \frac{x}{x-j1} = \frac{1}{R} \cdot \frac{x(x+j1)}{1+x^2}$$
$$= \frac{1}{R} \cdot \frac{x^2+jx}{1+x^2} \qquad (13)$$

Adding together the admittances *(6)* and *(13)*, the combined input admittance is

$$Y = Y_L + Y_H = \frac{1}{R} \cdot \frac{1-jx+jx+x^2}{1+x^2} = \frac{1}{R} \qquad (14)$$

whence the input impedance is

$$Z = R \qquad (15)$$

which is constant, resistive, and equal to both output loads.

When $x=1$, both P_L and P_H have half energy values, so this represents crossover frequency. Possible configurations using single-element filters are shown at *Fig.* 2. All reactances are equal to R at crossover frequency.

Two-Element Filters

The filter configurations for parallel connection at the input are shown at *Fig.* 3. For the low-pass section,

$$A_L = 1 + j\omega L\left(\frac{1}{R} + j\omega C\right)$$
$$= 1 - \omega^2 LC + j\frac{\omega L}{R} \qquad (16)$$

Using the same method as before, the energy supplied to the low-frequency unit will be

$$P_L = \frac{E^2}{R} \cdot \frac{1}{1 - 2\omega^2 LC + \omega^4 L^2C^2 + \omega^2 L^2/R^2} \qquad (17)$$

and the attenuation response is given by

$$db_L = 10 \log_{10} \left[1 - 2\omega^2 LC + \omega^4 L^2 C^2 + \frac{\omega^2 L^2}{R^2}\right] \qquad (18)$$

To conform with conditions *(i)* and *(ii)*, *(17)* and *(18)* must reduce to the forms

$$P_L = \frac{E^2}{R} \cdot \frac{1}{1+x^4} \qquad (19)$$

and

$$db_L = 10 \log_{10}[1+x^4] \qquad (20)$$

respectively, which require $L^2/R^2 = 2LC$ and $\omega^2 LC = x^2$. These equations are satisfied by making

$$\frac{\omega L}{R} = \sqrt{2}x \text{ and } \frac{1}{\omega CR} = \frac{\sqrt{2}}{x} \qquad (21)$$

Substituting these values into *(16)* gives

$$A_L = 1 - x^2 + j\sqrt{2}x \qquad (22)$$

Applying this to the phase response,

$$\phi_L = \tan^{-1}\frac{\sqrt{2}x}{1-x^2} \qquad (23)$$

and the input impedance of the low-pass unit is

$$Z_L = R\left(j\sqrt{2}x + \frac{\sqrt{2}}{\sqrt{2}+jx}\right)$$
$$= R\left(\frac{\sqrt{2}(1-x^2)+j2x}{\sqrt{2}+jx}\right) \qquad (24)$$

From which the admittance is

$$Y_L = \frac{1}{R}\left(\frac{\sqrt{2}+jx}{\sqrt{2}(1-x^2)+j2x}\right)$$
$$= \frac{1}{R}\left[\frac{2-j\sqrt{2}(x+x^3)}{2(1+x^4)}\right] \qquad (25)$$

Turning to the high-pass section,

$$A_H = 1 - \frac{1}{\omega^2 LC} + \frac{1}{j\omega CR} \qquad (26)$$

The energy supplied to the high-frequency unit will be

$$P_H = \frac{E^2}{R} \cdot \frac{1}{\left(1-\frac{1}{\omega^2 LC}\right)^2 + \left(\frac{1}{\omega CR}\right)^2}$$
$$= \frac{E^2}{R} \cdot \frac{\omega^4 L^2 C^2}{\omega^4 L^2C^2 - 2\omega^2 LC + 1 + \omega^2 L^2/R^2} \qquad (27)$$

and its attenuation response

$$db_H = 10 \log_{10} \left[1 - \frac{2}{\omega^2 LC} + \frac{1}{\omega^4 L^2 C^2} + \frac{1}{\omega^2 C^2 R^2}\right] \qquad (28)$$

which should take the form

$$P_H = \frac{E^2}{R} \cdot \frac{x^4}{1+x^4} \qquad (29)$$

and

$$db_H = 10 \log_{10} \left[\frac{1+x^4}{x^4}\right] \qquad (30)$$

requiring the same values as given in *(21)*.

Condition *(i)* is seen to apply by adding *(19)* and *(29)*:

$$P = P_L + P_H = \frac{E^2}{R} \cdot \frac{1+x^4}{1+x^4} = \frac{E^2}{R} \qquad (31)$$

Substituting the values of *(21)* into *(26)* gives

$$A_H = 1 - \frac{1}{x^2} - j\frac{\sqrt{2}}{x} \qquad (32)$$

This gives the phase response as

$$\phi_H = \tan^{-1}\frac{-\sqrt{2}x}{x^2-1}$$

which can be transposed to

$$\phi_H = \left(\tan^{-1}\frac{\sqrt{2}x}{1-x^2}\right) - \pi \qquad (33)$$

from which it is seen that the phase difference between *(23)* and *(33)* is constant at π or 180 deg.

The input impedance of the high-pass unit is

$$Z_H = R\left(\frac{j\sqrt{2}x}{1+j\sqrt{2}x} - j\frac{\sqrt{2}}{x}\right)$$
$$= R\left(\frac{2x + j\sqrt{2}(x^2-1)}{x(1+j\sqrt{2}x)}\right) \qquad (34)$$

and the admittance is

$$Y_H = \frac{1}{R}\left(\frac{x(1+j\sqrt{2}x)}{2x+j\sqrt{2}(x^2-1)}\right)$$
$$= \frac{1}{R}\left(\frac{2x^4 + j\sqrt{2}(x+x^3)}{2(1+x^4)}\right) \qquad (35)$$

Thus, by adding *(25)* and *(35)*, it is seen that the combined input admittance is $1/R$, as before, and the input impedance is equal to the load connected to each output and is always resistive.

Possible configurations using two element filters are shown at *Fig.* 4, with the reactance values at crossover frequency.

Three-Element Filters

Figure 5 shows the basic three-element filters for parallel combination at the input.

By similar development, it may be shown that the energy delivered to the low-frequency unit is

$$P_L = \frac{E^2}{R} \cdot \frac{1}{1+x^6} \qquad (36)$$

and the attenuation response is

$$db_L = 10 \cdot \log_{10}(1+x^6) \qquad (37)$$

respectively to fulfill conditions *(i)* and *(ii)*.

The phase response is given by

$$\phi_L = \tan^{-1}\frac{2x-x^3}{1-2x^2} \qquad (38)$$

The corresponding expressions for the high-pass unit are

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CONSTANT-RESISTANCE CROSSOVERS

(*from page 22*)

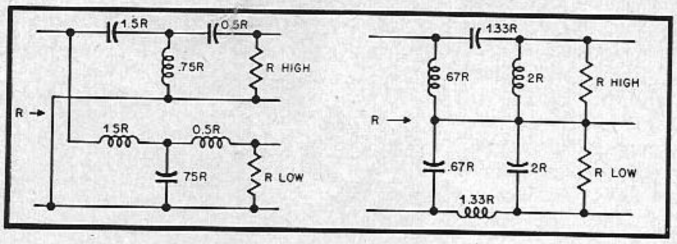


Fig. 6. Complete design data for the two constant-resistance arrangements using three-element sections.

$$P_H = \frac{E^2}{R} \cdot \frac{x^6}{1+x^6} \qquad (39)$$

$$db_H = 10 \log_{10} \left[\frac{x^6+1}{x^6}\right] \qquad (40)$$

$$\phi_H = \tan^{-1} \frac{2x^2-1}{x^3-2x}$$

$$= \left(\tan^{-1} \frac{2x-x^3}{1-2x^2}\right) - \frac{3\pi}{2} \qquad (41)$$

showing a constant phase difference of $3\pi/2$ or 270 deg.

Possible configurations using this relationship are shown at *Fig.* 6.

Practical Aspects

Figure 7 shows the response curves in the vicinity of crossover for each of these arrangements, with the addition of four-element filters, the derivations for which are not given. Stress is often placed on the slope of a crossover filter, the implication being that the steeper it is the better. Presumably this is a result of a desire to ensure that each unit handles only those frequencies for which it is intended, and that frequencies beyond its range are attenuated as much as possible. In the case of high-pass filters this can have good reason, because frequencies below the acoustic cut-off of a loudspeaker suffer distortion if permitted to pass, and can also injure the voice coil assembly.

But there are other aspects that deserve attention. How about the loudspeaker impedance characteristic? The filters are designed on the basis of feeding into a constant-resistance load at each output. The properties of these networks depends upon correct termination. Practical dynamic loudspeakers depart from this ideal in two ways:
(a) the motional impedance, near resonance particularly, contributes a rise in electrical impedance, similar to that of a parallel resonant circuit;
(b) the voice-coil inductance causes an inductive rise in impedance at the higher frequencies in the unit's range.

With some filter configurations, the latter effect can be compensated for, and the former effect probably takes place at frequencies well removed from crossover, and so will not affect the filter characteristic appreciably.

Voice-coil inductance can be compensated for by using a filter that has series inductance at the output end of the low-pass unit. The actual voice-coil inductance is subtracted from the nominal value of this inductance, given by the design data, and the filter inductance adjusted to pad the total inductance out to the design value.

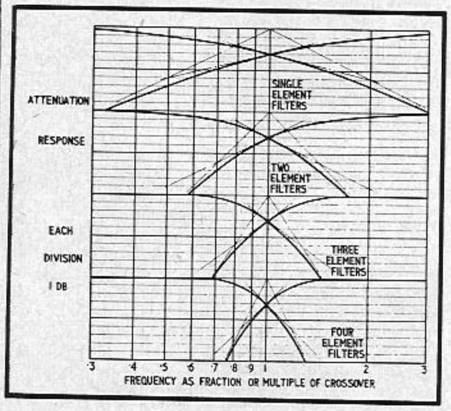


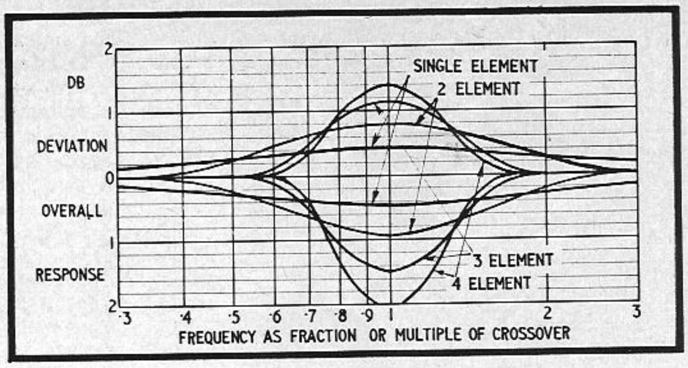
Fig. 7. Attenuation responses of composite filters in the vicinity of crossover. Construction lines show the slope at crossover, and the ultimate slope.

Assuming that such discrepancies are taken care of, what about production deviation from theoretical values? The larger the number of components required, the greater the possible number of combinations of error. Assume for simplicity that the errors on one filter are all the same way. Then *Fig.* 8 shows the deviation from level over-all response for each configuration, when the values are 10 per cent off nominal either way. It will be noted that the error not only produces greater deviation from zero level with the more complicated circuits, but also produces a more rapid change in level with frequency, which is known to give more coloration to reproduction than a gradual slope.

Confusion has sometimes arisen in defining the slope of a cut-off. The author prefers to designate a characteristic by its ultimate slope. Others seem to use the slope at the crossover point, which, for constant-resistance types, is just half the ultimate. These slopes are shown by construction lines in *Fig. 7,* and the following table lists the values in db/octave.

Number of elements per filter	Slope at crossover. (db/octave)	Ultimate slope. (db/octave)
1	3	6
2	6	12
3	9	18
4	12	24

Fig. 8. Showing the deviation in combined response from level, due to an error of 10 per cent, either way, in component values for each filter section.



The reason for greater and more sudden deviation with error in the more complicated filters is evident from the greater slope they possess at crossover. The steepness of the phase characteristics, shown in *Fig. 9*, also means that error can produce greater phase deviation between the two units with the more complicated filters.

Another feature about reproduction is its handling of transients. It is well known that there is an inherent limit to the sharpness of cut-off before transient distortion begins. But doesn't the use of complementary responses, so the overall result is flat, avoid this? Transient response is tied up with how a circuit handles wavefronts, as distinct from continuous waves. Although relative phase, within a cycle or two, does not matter in the reproduction of steady composite tones, it has considerable effect upon wavefronts. For faithful treatment of transients, all component frequencies should be passed on without any delay, or else with uniform delay.

The phase responses of the low-pass filters are shown in *Fig.* 9. The curve for the high-pass filter is exactly similar in each case, but advanced by the total ultimate phase change, which is the same as the constant phase difference noted in the foregoing analysis, in each case. Use of constant-resistance types ensures that there is no change in relative phase between the two outputs, because the difference is constant, but the *absolute* change of phase for both outputs together becomes more rapid at crossover as the number of elements is increased.

In the single-element filters the phase change is so gradual that decade of frequency is required to accommodate a change of 80 deg. The ultimate change is only 90 deg.

In two-element filters a phase change of 90 deg., from 45 deg. to 135 deg., occurs over a frequency ratio of 3.732, nearly two octaves.

The three-element type produces a phase change of 90 deg., from 90 deg. to 180 deg., in just an octave, while complete phase reversal, from 45 deg. to 225 deg., takes a frequency ratio of 6.85, between two and three octaves.

The four-element filter produces 90 deg. phase change in a frequency ratio of only 1.38, less than half an octave; 180 deg. phase change, from 90 deg. to 270 deg., takes a frequency ratio of 3.08, between one and two octaves, while it passes through three right angles of its ultimate four in little over a decade.

Visualize what happens to a composite signal using four-element filters. At crossover, both units must be acoustically near enough in phase, to avoid undesirable dissociation or cancellation effects in this vicinity. The ultimate low- and high-frequency energy from their respective units will be in phase, but both in anti-phase with energy from both units at crossover. Taking in the whole spectrum view, then, the whole band is in phase except for a range of frequencies near crossover in which phase reversal takes place. So a wavefront composed of a wide range of frequencies will probably have just one of those frequencies completely reversed in phase. If the initial transient is free from coloration, reversal of this one frequency will produce a pronounced ripple on the wavefront at its own frequency, making the transient noticeably colored. This will happen, although steady-tone response of the composite arrangement may be as straight as a line drawn with a ruler.

Choice of filter arrangement will depend on application. If both loudspeaker units have a good response, and overlap somewhat, there is no need whatever to have a filter using more than one element in each section. If each unit is listened to separately, it may seem that

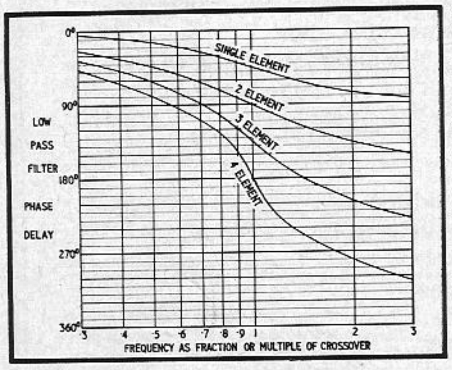


Fig. 9. Comparison of the phase response for filters using from one to four elements. Response for the low-pass section is shown. That for the high-pass section is similar.

the "die" beyond crossover is rather too gradual. But listen to the two units together. If their sensitivities are about equal, even with single-element filtering, the frequency does not have to be far removed from crossover for a very definite impression to be formed as to which unit is delivering apparently all the sound. The unit receiving the greater proportion of energy at a particular frequency will always mask the other one, provided the other is not producing any distortion.

For cases where the acoustic cut-off necessitates more rapid reduction in energy fed to the high-frequency unit, the two-element filter will usually be adequate. It is true that acoustic cut-off can exceed the ultimate slope of 12 db/octave, but this aspect may be safeguarded by moving crossover a little farther from the acoustic cut-off, so as to allow a margin. This will also ensure that signal amplitudes sufficient to damage the h.f. unit (below cut-off) do not reach it.

Intermodulation

As well as providing more uniform coverage over a wide range, one of the advantages of using separate units for different parts of the frequency spectrum is the avoidance of intermodulation distortion. In its commonest form, this is the modulation of h.f. tones by large amplitude l.f. tones, producing a "dithery" effect in the reproduction. By feeding the l.f. tones to a large unit designed to handle them comfortably, and the h.f. ones to another equipped specifically for responding to their more rapid vibrations, intermodulation distortion due to loudspeakers is largely avoided.

While loudspeakers can introduce much more intermodulation distortion than a respectable amplifier, no amplifier is completely free from this form of distortion. For this reason, some experimenters prefer to separate the frequency bands before final power amplification, using an individual power amplifier to drive each loudspeaker unit. Channel filters of this type should have attenuation and phase responses similar to the crossovers used in loudspeaker circuits, but the impedance at which they operate will be quite different. They can more readily be correctly terminated, and efficiency is not so important, as they will not be absorbing precious output power.

Filters for this application use more convenient capacitor values, but the inductance values come out rather large for high-impedance circuits. Inductances for use at low levels are also prone to pick up supply ripple or hum. Both these objections may be overcome by using negative feedback in conjunction with suitable capacitor values. Design for circuits of this type to give attenuation and phase responses with the same properties as those developed in this article, will be presented in a future article.