A New Approach to Negative Feedback Design

N. H. CROWHURST*

A thorough discussion of the characteristics of individual amplifier stages and their relation to the over-all performance of a feedback amplifier.

SINCE THE APPEARANCE of the author's handbook "Feedback," in which appeared for the first time some charts specially prepared to aid in working out design details, several friends and correspondents have suggested that the basis for these charts should be published. Most people find difficulty in digesting the mathematics of design, for which reason such details were deliberately left out of the handbook. However, further work since preparing the material in the book promises to lead to interesting new developments in the design of feedback amplifiers, and for this reason it would seem to be time to publish a little more about the method.

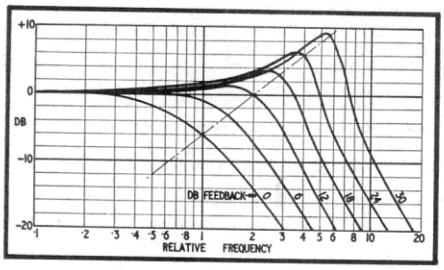

Fig. 1. Showing the effect of feedback on a feedback loop containing two identical stages. All curves plotted to the same zero reference level.

When a number of stages are connected into a closed loop, possibility of instability, or the consideration of frequency response of the combination, is concentrated in two principal components, contiguous with the low- and high-frequency cutoffs of the arrangement respectively. Series capacitor elements, in the interstage couplings, and any shunt inductors contribute towards the low-frequency cutoff of the complete arrangement, while the interstage shunt capacitance (to ground), and any series inductance (such as transformer leakage inductance), contribute to the high-frequency cutoff.

The simplest way to designate the characteristic of a single element producing a 6 db/octave cut-off in either direction is by its time constant, as this avoids the necessity for calculating the reactance of capacitances and inductances at different frequencies and also yields a more direct approach at a later stage. For the purposes of this treatment, each stage is assumed to possess a single reactance causing low-frequency cutoff and a single reactance causing high-frequency cutoff. It is also assumed that no interaction occurs between the impedances of successive stages other than around the complete loop, and that there is no appreciable interaction between the components of the stage causing cutoff at the opposite ends of the frequency spectrum. Where such interaction does in fact occur, the treatment is usually only modified quantitatively, although in some cases, particularly where transformers are included in the loop, some of the time constants theoretically become complex quantities. This does not complicate matters as much as may be expected, because the necessity for actually evaluating complex time constants is avoided in this method, as will be shown later. In application, the number of equivalent stages around the loop for l.f. and h.f. cutoff representation may not always be identical.

General Form

To pave the way for detailed treatment, the h.f. response of a single network can be represented by the expression $1 + jx$, where $x = f/f_o$, and f_o is the frequency where the shunt reactance is equal to the circuit resistance it shunts. A number of such responses combined, but not necessarily using the same f_o, can be represented, with respect to a suitable reference frequency, by an equation,

$$D = 1 - ax^2 + bx^4 \ldots + jcx - jdx^3 + jex^5 \ldots \quad (1)$$

This expression represents the loss due to these couplings in both magnitude and phase. A similar expression can represent the l.f. response by using $x = f_o/f$.

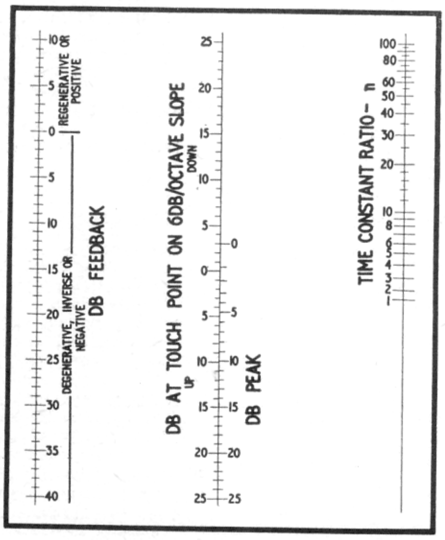

Fig. 3. An abac to aid in calculating the response of any feedback loop with two stages, using positive or negative feedback.

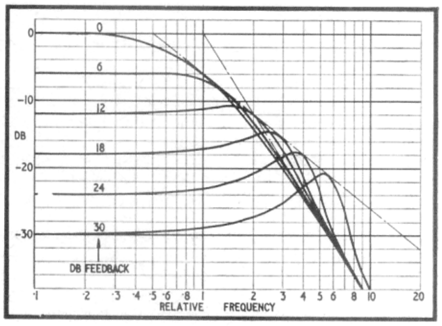

Fig. 2. The curves of Fig. 1 replotted to take loss of gain due to feedback into account. The significance of the chain dotted lines in these figures is explained in the text.

Assume now that an amplifier has a gain, where no reactances are having any effect, of A_m; then the gain at other points will be given by

$$A = \frac{A_m}{D} \quad (2)$$

Now we introduce the well-known feedback equation, using A_{fm} to represent the gain with feedback at a frequency where no reactances are having effect,

$$A_{fm} = \frac{A_m}{1 + A_m\beta} \quad (3)$$

or at other frequencies,

$$A_f = \frac{A}{1 + A\beta} \quad (4)$$

Substituting Eq. (2) into this gives

$$A_f = \frac{A_m}{D + A_m\beta} \quad (5)$$

This can be rearranged to give the effective attenuation from mid-band gain (without feedback),

$$D_f = \frac{A_m}{A_f} = D + A_m\beta \quad (6)$$

In expression (3), $A_m\beta$ is the loop gain (or loss, but usually greater than unity, representing a gain) and $1 + A_m\beta$ is the feedback factor, by which gain is modi-

* *82, Canterbury Grove, London, S.E. 27, England.*

fied, as well as impedance, distortion and anything else for which feedback may be used. Writing, for the feedback factor, $F = 1 + A_m\beta$, and substituting *(1)* into *(6)*, the latter may be re-written.

$$D_f = F - ax^2 + bx^4 \ldots \quad jcx - jdx^3 + jex^5 \ldots \qquad (7)$$

the right side of which is identical to that of *(1)*, except that F has been substituted for *1*. This fact proves convenient in developing the expressions for various conditions.

Single Stage Loop

Applying this to the simple single-stage case, where the feedback loop only includes one reactance affecting cutoff at the h.f. end (or similarly for the l.f. end),

$$D = 1 + jx \qquad (8)$$

and

$$D_f = F + jx \qquad (9)$$

In this case the 3-db loss point, which is also the frequency at which phase shift is 45 deg., occurs where the imaginary term is equal to the real term. Without feedback this is when $x = 1$. With feedback, as shown by *(9)*, it is when $x = F$. This means that for this case the frequency range is extended in direct proportion to F, the feedback factor.

Two-Stage Loop

Consider first the case using two couplings with identical h.f. cutoff characteristic, for which

$$D = (1 + jx)^2 = 1 - x^2 + j2x \qquad (10)$$

and

$$D_f = F - x^2 + j2x \qquad (11)$$

Squaring both sides, and taking 10 times the logarithm to the base 10, the expression for db response becomes,

$$db = 10 \log_{10} D_f^2 = 10 \log_{10}[(F - x^2)^2 + 4x^2] = 10 \log_{10}[F^2 + (4 - 2F)x^2 + x^4] \qquad (12)$$

Differentiating the term in brackets with respect to x and equating to zero will find the location of any peak in the response. This gives

$$x_p^2 = F - 2 \qquad (13)$$

From this it is evident that there is no peak provided $F < 2$, or 6 db feedback. For values of F greater than 2, the square root of expression *(13)* gives the frequency of peak in terms of the original cutoff frequency of each network as reference. Peak height is given by substituting *(3)* and *(13)* into *(12)*,

$$db_p = 10 \log_{10} \frac{F^2}{D_f^2} = 10 \log_{10} \frac{F^2}{4(F-1)} \qquad (14)$$

For large values of feedback, this approaches $10 \log_{10} (F/4)$, which means that 2:1 increase in feedback (6 db) then raises the peak height by an additional 3 db.

Another reference of particular interest in this case is the point where the response slope is 6 db/octave. This is found by equating $\frac{d \log D_f}{d \log x} = 1$, which gives

$$\frac{d \log D_f}{d \log x} = \frac{d\, 2 \log D_f}{dD_f^2} \times \frac{dD_f^2}{dx^2} \times \frac{dx^2}{d\, 2 \log x} = \frac{2x^4 - 2(F-2)x^2}{x^4 - 2(F-2)x^2 + F^2} = 1$$

which simplifies to give an expression for the 6 db/octave slope frequency,

$$x_6^2 = F \qquad (15)$$

Attenuation at x_6, the 6 db/octave slope point, is given by substituting *(3)* and *(15)* into *(12)*, giving,

$$db_6 = 10 \log_{10} \frac{F^2}{D_f^2} = 10 \log_{10} \frac{4}{F} \qquad (16)$$

Table I gives a comparison of responses at intervals of 6 db feedback. Positive db figures represent attenuation; negative, lift.

TABLE I

Feedback		6 db/octave pt		Peak	
db	F	x_6^2	db_6	x_p^2	db_p
0	1	1	+6	–	–
6	2	2	+3	–	–
12	4	4	0	2	–1.25
18	8	8	–3	6	–3.6
24	16	16	–6	14	–6.3
30	32	32	–9	30	–9.17

Figure 1 shows this family of curves plotted with a common zero reference level. At *Fig.* 2 the same curves are drawn to take into account the loss of gain due to feedback. From this it appears that the 6 db/octave slope point is always tangential to a 6 db/octave line passing through zero level at half the cutoff frequency of both circuits. On the common zero reference level presentation of *Fig.* 1, these points fall on a rising line of 6 db/octave slope. In *Fig.* 2, the ultimate cutoff is the same 12 db/octave response (also shown by a chain dotted line). These two constructions with this presentation help in visualizing how the response changes as feedback is progressively increased.

In this two-stage case, the response never becomes unstable, a condition that is indicated by infinite peak height.

The foregoing has applied to identical cutoff networks combined. In practice many other combinations can occur. It will be assumed that one network has n times the time constant of the other. So

$$D = (1 + jx)(1 + jnx) = 1 - nx^2 + j(n+1)x \qquad (17)$$

and

$$D_f = F - nx^2 + j(n+1)x \qquad (18)$$

and the response is

$$db = 10 \log_{10} D_f^2 = 10 \log_{10}[F^2 + \{(n+1)^2 - 2Fn\} x^2 + n^2x^4] = 10 \log_{10}[(F - nx^2)^2 + (n+1)^2x^2] \qquad (19)$$

Differentiating with respect to x and equating to zero gives

$$x_p^2 = \frac{F}{n} - \frac{(n+1)^2}{2n^2} \qquad (20)$$

Substituting this, with *(3)*, into *(19)* gives peak height as

$$db_p = 10 \log_{10} \frac{F^2}{D_f^2} = 10 \log_{10} \times \frac{F^2}{\frac{(n+1)^2}{n} F - \frac{(n+1)^4}{4n^2}} \qquad (21)$$

To find the 6 db/octave slope reference point:

$$\frac{d \log D_f}{d \log x} = \frac{2n^2x^4 - [2Fn - (n+1)^2]x^2}{n^2x^4 - [2Fn - (n+1)^2]x^2 + F^2} = 1$$

or

$$x_6^2 = \frac{F}{n} \qquad (22)$$

Whence attenuation at the 6 db/octave slope point is

$$db_6 = 10 \log_{10} \frac{(n+1)^2}{nF} \qquad (23)$$

From *(20)* it is evident that there is no peak provided

$$F < \frac{(n+1)^2}{2n}$$

Substituting this limiting value of F into *(23)* gives the attenuation at the 6 db/octave slope point as $10 \log_{10} 2$, or 3 db. The factor $(n+1)^2/4n$ is important, because it represents the effect of staggering the time constants by the ratio n on the response shaping. For this reason terms including this factor appear in expressions *(20)*, *(21)*, and *(23)*.

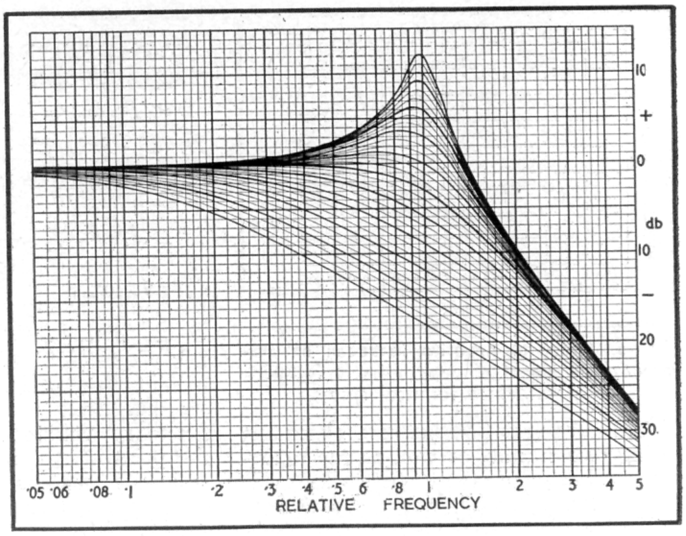

Fig. 4. Variations in response shaping possible with two-stage loops. The frequency scale is relative to the touch point on a 6 db/octave slope.

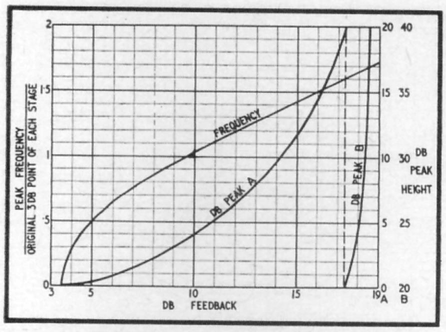

Fig. 5. Effect of negative feedback on frequency and height of peak, using a feedback loop with three identical stages.

This inter-relation between quantities using two cutoffs, as well as the inherent stability of these networks, will prove useful in designing feedback amplifiers with desired correction characteristics and rock-steady stability. Another useful fact about two-stage cutoffs is that the half-phase-shift of 90 deg. occurs at the 6 db/octave point.

Since the basic variables are so few, a simple three line abac can tell all there is to know about these networks, shown at *Fig.* 3. This gives, for db feedback on the left and time-constant ratio n on the right, the shape of response applicable in *Fig.* 4, which is plotted with the 6 db/octave slope point as reference.

This information is also applicable to the response of a.f. transformers, as appears from the fact that *Fig.* 4 is actually the same as *Fig.* 2 of the article "Making the Best of an Audio Transformer" in the January, 1953, issue. Conditions with the 6 db/octave slope point above a level of 6 db below zero level, without feedback, will be represented on the abac of *Fig.* 3 by points on the Time Constant Ratio scale below $n=1$, which is left a blank line. This region represents complementary complex time constants, but their exact value is unimportant, because the appropriate point on *Fig.* 3 can be used to see the effect of any degree of feedback.

To use the information in the abac for such cases, the response curve of the transformer in its associated circuit is taken, and either the height of the peak or the 6 db/octave touch point noted. For the latter, which must be used when there is no peak, the response is plotted on db/log-frequency paper, and a 6 db/octave slope is drawn touching the response curve. The attenuation below or above zero reference level at this touch point is noted and used on the chart of *Fig.* 3. Provision is also made on this abac for positive feedback prediction, up to 10 db. This can prove useful for eliminating the peak in the response of transformer coupled circuits, using the kind of feedback for the required impedance effect as well.

Output source impedance is reduced by negative voltage feedback, or positive current feedback. Conversely it is increased by positive voltage feedback or negative current feedback. *An advantage of the positive variety of feedback in this connection is that zero or infinite impedance can be achieved quite simply with absolute stability.*

Three-Stage Loops

Taking first the case using three couplings with identical time constants:

$$D=(1+jx)^3=1-3x^2+j3x-jx^3 \quad (24)$$

$$D_f=F-3x^2+j3x-jx^3 \quad (25)$$

$$db=10\log_{10}D_f^2=10\log_{10}\times[F^2+(9-6F)x^2+3x^4+x^6] \quad (26)$$

$$x_p^2=\sqrt{2(F-1)}-1 \quad \text{(only real root)} \quad (27)$$

$$db_p=10\log_{10}\left[\frac{F^2}{(F-1)[F+7-4\sqrt{2(F-1)}]}\right] \quad (28)$$

With three-stage networks there is a stability limit to F, so there are two boundary conditions of interest: (a) the point at which peaking commences, and (b) the point where instability commences. The former occurs in h.f. cutoffs where the peak frequency passes through zero, before becoming imaginary. For l.f. cutoffs the peak frequency passes through infinity (i.e. $x_p=0$ in either case). From *(27)* this is at $F=1.5$, or 3.522 db feedback. The latter boundary occurs at a point where D_f

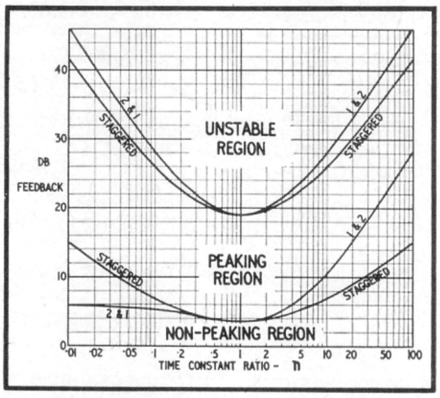

Fig. 6. Limit chart to aid in assessing performance of three-stage loops with non-identical time constants.

becomes zero, for which both its real and imaginary parts must be zero. Equating the imaginary part to zero finds the value of x^2 at which it occurs, and then substituting this value in the real part finds the value of F. For three identical h.f. cutoffs instability occurs at $x_s^2=3$, or $x_s=\sqrt{3}$, and $F_s=9$, or 19.1 db.

The half-slope point could be found by equating $\frac{d\log D_f}{d\log x}=\frac{3}{2}$, but this does not have the same usefulness as in the two-stage case.

Turning to non-identical cases, which

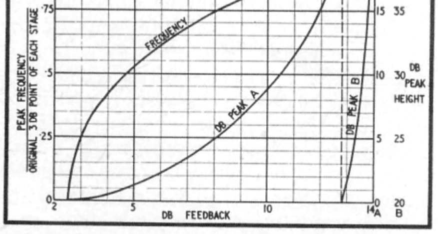

Fig. 7. Effect of negative feedback on peak frequency and height, using a feedback loop with four identical stages.

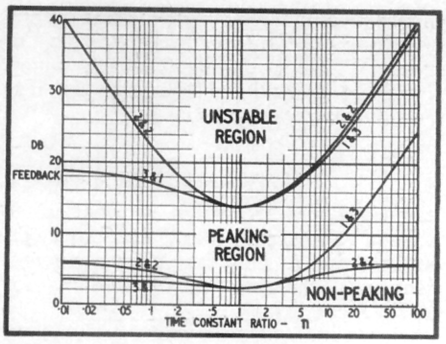

Fig. 8. Limit chart to aid in assessing performance of four-stage loops with non-identical time constants.

are necessary for practical application, the time constants can vary in more ways than where there are only two networks. Extreme possibilities can be represented by using n for the ratio between the time constants having the widest difference, and then considering (a) the case of two at one extreme and one at the other, and (b) the case of three networks geometrically staggered within this range. Every other possibility must fall between these extremes.

One and Two

Assuming one time constant is n times each of the other two (for h.f. cases; for l.f. cases, the same formulas will apply by using $1/n$ times the other two):

$$D=(1+jx)^2(1+jnx)=1-(2n+1)x^2+j(2+n)x-jnx^3$$

$$D_f=F-(2n+1)x^2+j(2+n)x-jnx^3 \quad (29)$$

$$db=10\log_{10}[F^2+\{(2+n)^2-2F(2n+1)\}x^2+(2n^2+1)x^4+n^2x^6] \quad (30)$$

Here it is evident that the peaking boundary can be found by equating the x^2 coefficient to zero, or

$$F_p=\frac{(2+n)^2}{2(2n+1)} \quad (31)$$

As before the boundary for stability is found by equating both parts of D_f to zero, giving

$$x_s^2=\frac{2+n}{n} \text{ and } F_s=\frac{(2+n)(2n+1)}{n}=\frac{2n^2+5n+2}{n} \quad (32)$$

Staggered

Here the extreme time constants can be assumed each to have a ratio of $n^{1/2}$ to the central one, in opposite directions.

$$D=(1+jn^{-1/2}x)(1+jx)(1+jn^{1/2}x)=1-(n^{-1/2}+1+n^{1/2})x^2+j(n^{-1/2}+1+n^{1/2})x-jx^3 \quad (33)$$

$$D_f=F-(n^{-1/2}+1+n^{1/2})x^2+j(n^{-1/2}+1+n^{1/2})x-jx^3 \quad (34)$$

$$db=10\log_{10}[F^2+\{(n^{-1/2}+1+n^{1/2})^2-2F(n^{-1/2}+1+n^{1/2})\}x^2+(n^{-1/2}+1+n^{1/2})x^4+x^6] \quad (35)$$

The peaking boundary occurs where the x^2 coefficient is zero, or

$$F_p=\frac{n^{-1/2}+1+n^{1/2}}{2} \quad (36)$$

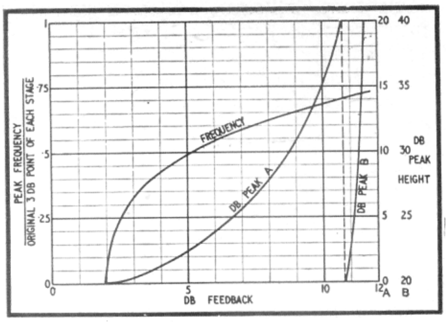

Fig. 9. Effect of negative feedback on peak frequency and height, using a feedback loop with five identical stages.

and stability boundary where both parts of $D_f = 0$, or

$$x_s^2 = n^{-1/2} + 1 + n^{1/2} \text{ and } F_s = (n^{-1/2} + 1 + n^{1/2})^2 \quad (37)$$

For three-stage networks, *Fig.* 5 shows a plot of expressions *(27)* and *(28)* for identical networks, and *Fig.* 6 a plot of expressions *(31)*, *(32)*, *(36)*, and *(37)* for non-identical loops. *Figure* 5 gives an idea of the rate at which transition from one boundary to the other occurs, while *Fig.* 6 shows the boundaries for limiting cases, using a maximum time constant ratio of n. Fractional values of n mean that the two similar time-constant cutoffs come into action before the remaining one, and vice versa with values greater than unity. With the staggered arrangement the curves are obviously symmetrical for both boundaries. For the stability boundary they are both symmetrical, but the one and two arrangement gives the highest peaking boundary for values of n greater than unity, that is, when one network introduces cutoff acting nearer the pass range than the other two.

Four Stage Loops

Taking first the case using four couplings with identical time constants:

$$D = (1+jx)^4 = 1 - 6x^2 + x^4 + j4x - j4x^3 \quad (38)$$

$$D_f = F - 6x^2 + x^4 + j4x - j4x^3 \quad (39)$$

$$db = 10 \log_{10}[F^2 + (16 - 12F)x^2 + (4 + 2F)x^4 + 4x^6 + x^8] \quad (40)$$

To find the peak conditions, the expression in square brackets is differentiated with respect to x^2 and equated to zero, leading to the expression,

$$x^6 + 3x^4 + 2x^2 + 4 = F(3 - x^2).$$

This is a cubic equation in x^2. To plot the frequency of peak, it is simpler to take the frequency as independent variable and then find corresponding values of F from,

$$F = \frac{x_p^6 + 3x_p^4 + 2x_p^2 + 4}{3 - x_p^2} \quad (41)$$

To know the limits between which to plot, the value of x^2 producing instability is $x_s^2 = 1$, and as before, peaking commences at $x^2 = 0$.

To find the height of the peak, still using x as independent variable, values of F from *(41)* are substituted into *(40)*. The results are plotted in *Fig.* 7, using F as the common variable for convenience.

The peaking boundary is $F_p = 4/3$, or 2.5 db feedback, and the stability boundary is given by equating both parts of *(39)* to zero, whence,

$$x_s = 1 \text{ and } F_s = 5, \text{ or 14 db feedback} \quad (42)$$

For arrangements other than identical, still greater range is possible than for the three-stage case, but it is obvious that any staggered arrangement will not give such good possibilities as an arrangement using networks each of which is at one or other limit of the time-constant range, so such limits only need be considered. This reduces the number of possibilities to be presented to two.

One and Three

Assuming one time constant is n times each of the other three,

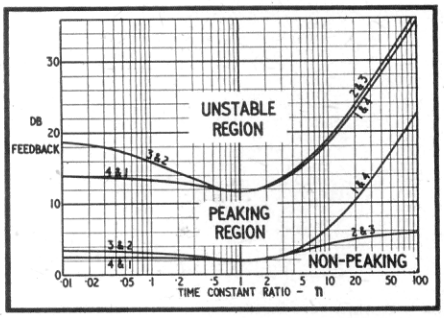

Fig. 10. Limit chart to aid in assessing performance of five-stage loops with non-identical time constants.

$$D = (1+jx)^3(1+jnx) = 1 - 3(1+n)x^2 + nx^4 + j(3+n)x - j(3n+1)x^3 \quad (43)$$

$$D_f = F - 3(1+n)x^2 + nx^4 + j(3+n)x - j(3n+1)x^3 \quad (44)$$

$$db = 10 \log_{10}[F^2 + \{(n+3)^2 - 6F(n+1)\}x^2 + \{3n^2 - 2n + 3 + 2nF\}x^4 + (3n^2+1)x^4 + n^2x^8] \quad (45)$$

Although it contains a negative term, the whole x^4 coefficient can never be negative, so the only possibility of a peak is when the x^2 coefficient is negative, whence the peaking boundary, as before, occurs when the x^2 coefficient is zero, or

$$F_p = \frac{(n+3)^2}{6(n+1)} \quad (46)$$

and the stability boundary is given by equating both parts of *(43)* to zero, whence,

$$x_s^2 = \frac{n+3}{3n+1}$$

$$\text{and } F_s = \frac{(n+3)(8n^2+9n+3)}{(3n+1)^2} \quad (47)$$

Two and Two

Assuming two pairs of identical time constants, of ratio n between pairs,

$$D = (1+jx)^2(1+jnx)^2 = 1 - (n^2+4n+1)x^2 + n^2x^4 + j2x(n+1)(1-nx^2) \quad (48)$$

$$D_f = F - (n^2+4n+1)x^2 + n^2x^4 + j2x(n+1)(1-nx^2) \quad (49)$$

$$db = 10 \log_{10}[F^2 + \{4(n+1)^2 - 2(n^2+4n+1)F\}x^2 + \{2n^2F + (n^2+1)^2\}x^4 + 2n^2(n^2+1)x^6 + n^4x^8] \quad (50)$$

The only possibility of a peak is when the x^2 coefficient is negative, so the peaking boundary is found by equating this coefficient to zero, or

$$F_p = \frac{2(n+1)^2}{n^2+4n+1} \quad (51)$$

and the stability boundary by

$$x_s^2 = \frac{1}{n} \text{ and } F_s = n + 3 + \frac{1}{n} \quad (52)$$

Curves of expressions *(46)*, *(47)*, *(51)*, and *(52)* are plotted in *Fig.* 8. Naturally the two-pairs arrangements has symmetrical curves. The 3-and-1 combination (three acting before one) has lower boundaries than any other

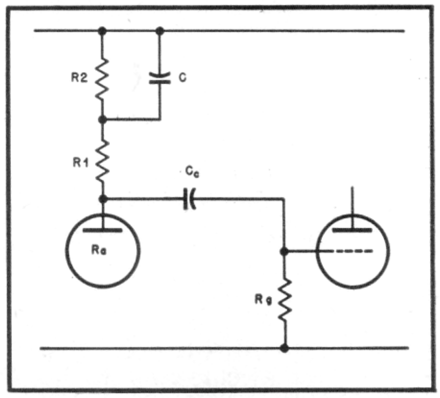

Fig. 11. This form of step circuit is often used in long over-all loops with large feedback, to aid in obtaining stability.

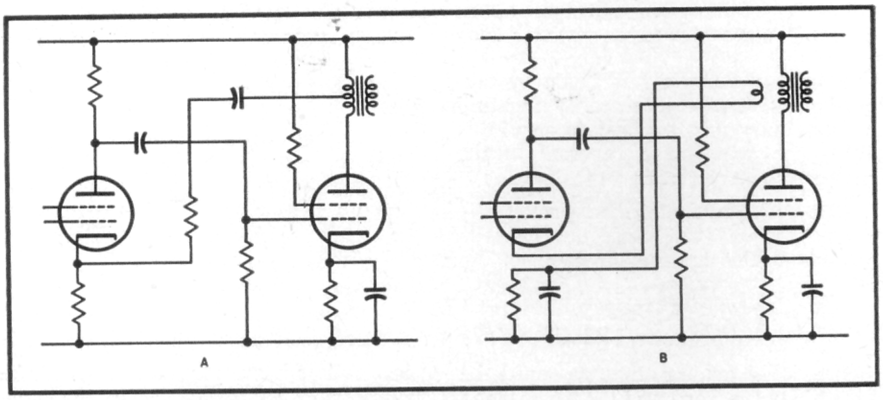

Fig. 12. Practical types of circuit for two-stage loops with ample feedback. These can be applied equally well to push-pull circuits, but are shown single-ended for simplicity.

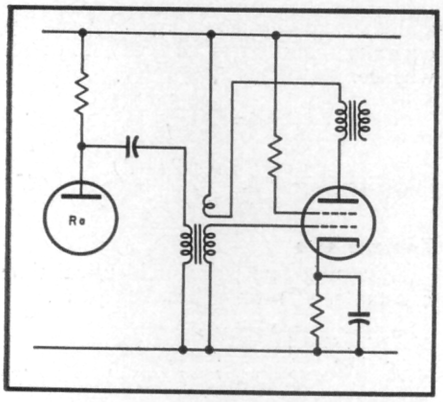

Fig. 13. This is a useful circuit for obtaining positive current feedback. With the cooperation of transformer manufacturers, it should feature in future amplifier circuits.

combination (three acting before one) has lower boundaries than any other combination of four cutoffs, and so is not of practical value unless instability is sought. Notice here that, though the two-and-two arrangement only approaches 6 db feedback before peaking occurs, however large n is made, so the one-and-three arrangement is better for minimizing peaking, the two-and-two arrangement is slightly better for its stability margin. The lesson here would seem to be that at least two of the networks should be removed beyond the range by a factor n, and the other two may have one at the nearer limit, and one somewhere between the first and second time-constant limits, dependent upon whether exact shape of response or margin of stability is regarded as the more important factor in design.

Five-Stage Loops

Taking first the case using networks with identical time constants:

$$D = (1+jx)^5 = 1 - 10x^2 + 5x^4 + j5x - j10x^3 + jx^5 \qquad (53)$$

$$D_f = F - 10x^2 + 5x^4 + jx - j10x^3 + jx^5 \qquad (54)$$

$$db = 10\log_{10}[F^2 + (25 - 20F)x^2 + 10Fx^4 + 10x^6 + 5x^8 + x^{10}] \qquad (55)$$

The peaking boundary is given by equating the x^2 coefficient to zero, or $F = 5/4$, that is 1.938 db.

Using the same method as for four-stage loops for relating feedback to peak frequency, and height,

$$F = \frac{x_p^8 + 4x_p^6 + 6x_p^4 + 5}{4(1 - x_p^2)} \qquad (56)$$

The stability boundary is given by taking the lowest root obtained by equating the imaginary part of *(54)* to zero, this giving the first phase reversal in the transfer characteristic,

$$x_s^2 = 5 - 2\sqrt{5} = 0.528 \text{ approx.}$$

or

$$x_s = 0.7266 \text{ approx.}$$

and

$$F_s = 80\sqrt{5} - 175 = 3.885 \text{ approx. or } 11.8 \text{ db} \qquad (57)$$

whence it is evident that x_p must be plotted between zero and 0.7266 in *(56)* to find values of F. Substituting these values into *(55)* gives the height of peak to correspond. *Figure* 9 shows these results.

Again taking two possibilities for the non-identical networks:

One and Four

Assume one network has a time constant n times the other four:

$$D = (1+jx)^4(1+jnx) = 1 - 2(3+2n)x^2 + (1+4n)x^4 + j(4+n)x - j2(2+3n)x^3 + jnx^5 \qquad (58)$$

$$D_f = F - 2(3+2n)x^2 + (1+4n)x^4 + j(4+n)x - j2(2+3n)x^3 + jnx^5 \qquad (59)$$

$$db = 10\log_{10}[F^2 + \{(4+n)^2 - 4(3+2n)F\}x^2 + \{4(n-1)^2 + 2F(1+4n)\}x^4 + 6n^2x^6 + (1+4n^2)x^8 + n^2x^{10}] \qquad (60)$$

From which the peaking boundary is given by

$$F_p = \frac{(4+n)^2}{4(3+2n)} \qquad (61)$$

and the stability boundary by

$$x_s^2 = 3 + \frac{2}{n} - 2\sqrt{2 + \frac{2}{n} + \frac{1}{n^2}}$$

and

$$F_s = 8\left(5n + 4 + \frac{1}{n}\right)\sqrt{2 + \frac{2}{n} + \frac{1}{n^2}} - \left(56n + 71 + \frac{40}{n} + \frac{8}{n^2}\right) \qquad (62)$$

Two and Three

Assume two networks each have a time constant n times that of the other three:

$$D = (1+jx)^3(1+jnx)^2 = 1 - (3+6n+n^2)x^2 + (2n+3n^2)x^4 + j(3+2n)x - j(1+6n+3n^2)x^3 + jn^2x^5 \qquad (63)$$

$$D_f = F - (3+6n+n^2)x^2 + (2n+3n^2)x^4 + j(3+2n)x - j(1+6n+3n^2)x^3 + jn^2x^5 \qquad (64)$$

$$db = 10\log_{10}[F^2 + \{(3+2n)^2 - 2(3+6n+n^2)F\}x^2 + \{(3-4n+n^4) + 2(2n+3n^2)F\}x^4 + (1+6n^2+3n^4)x^6 + (2n^2+3n^4)x^8 + n^4x^{10}] \qquad (65)$$

From which the peaking boundary is given by

$$F_p = \frac{(3+2n)^2}{2(3+6n+n^2)} \qquad (66)$$

and the stability boundary by

$$x_s^2 = \frac{3}{2} + \frac{3}{n} + \frac{1}{2n^2} - \sqrt{\frac{9n^2}{4} + 7n + \frac{15}{2} + \frac{3}{n} + \frac{1}{4n^2}}$$

and

$$F_s = \left(8n + 18 + \frac{12}{n} + \frac{2}{n^2}\right) \times \sqrt{\frac{9n^2}{4} + 7n + \frac{15}{2} + \frac{3}{n} + \frac{1}{4n^2}} - \left(12n^2 + 45n + 63 + \frac{42}{n} + \frac{12}{n^2} + \frac{1}{n^3}\right) \qquad (67)$$

Curves of expressions *(61)*, *(62)*, *(66)*, and *(67)* are plotted in *Fig.* 10, for values from .01 to 100, as in the other cases. Conclusions to be drawn from this are that three of the networks should have time constants to remove their cutoffs well beyond the frequency range, by a ratio n, while the remaining two may be adjusted according to the frequency response and margin of stability required.

Step Networks

Figure 11 shows a popular type of circuit often included in an over-all feedback loop to improve stability with large amounts of feedback. The same circuit may be applied for instability at either end of the response, using values suitable for the application. To apply this network in relation to the data here given, the simplest way is to regard the circuit as a synthesis of two time constants. The effect of one of these is inverted and would, if exactly equal to another somewhere else in the loop, cancel its effect, leaving the remaining time constant of the step circuit, operative at a higher frequency, in its place. The advantage of this method for improving h.f. stability is that less gain has to be sacrificed over the pass band in order to get the required time constant relationships, the effective plate coupling being $R_1 + R_2$ instead of just R_1. Applied for l.f. stability, one cutoff is brought into the pass band, but its effect is offset by the feedback; this saves the necessity for unduly large capacitors to obtain the time constants needed by the straight circuits.

Margin of Stability

It is often not appreciated that input and output impedances interact with the feedback in over-all feedback loop amplifiers. For example, where negative voltage feedback is used, the amount of feedback increases as the load impedance is raised. Similarly at the input end, where an input transformer is used particularly, the amount of feedback occurring at high frequencies will influence the response of the transformer, by modifying the impedance it "looks into" (This is assuming that the transformer itself does not form part of the feedback loop, i.e. feedback is injected in the grid circuit). This accounts for the fact that amplifiers with wonderful characteristics often exhibit unpleasant peaky effects when connected to certain

(Continued on page 53)

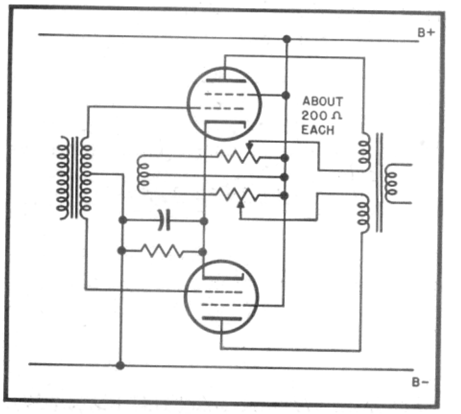

Fig. 14. Method of obtaining adjustment of positive current feedback, using the basic circuit of Fig. 13 in push-pull arrangement.

NEGATIVE FEEDBACK

(from page 30)

combinations of input and output circuits, even though the nominal impedances are all correct.

This effect can be divided into two parts: (a) the effect of external impedances on the characteristics of the feedback amplifier; and (b) the effect of the impedances presented by the feedback amplifier to the input and output circuits. From the present viewpoint, the former is the more important, and usually has the bigger effect. The feedback is calculated on the basis of constant resistances for input and output impedances, and with correct values of this kind the amplifier gives a wonderful response characteristic; but with a practical dynamic loudspeaker connected to the output, the load characteristic is quite different from a constant resistance, and the feedback loop may well be approaching its stability boundary, resulting in a pronounced peak in the response. Some amplifiers of this type confirm this fact by going into oscillation when the output load is disconnected altogether.

The author contends for this reason that a practical requirement for a good amplifier should be that it is completely stable, working into any load from open circuit to short circuit. This does not mean that it should be expected to deliver full undistorted output into impedances widely divergent from the nominal value. The nominal impedance should be within reasonable limits from the correct value, and then the inevitable deviations from nominal in the loudspeaker *impedance* frequency response (not to be confused with the loudspeaker's *acoustic* response) will not be likely to cause excessive variation in the amplifier from its nominal frequency characteristics.

This requirement would be difficult to meet, using large amounts of over-all feedback. For this reason the author recommends that feedback be taken over a shorter loop, including not more than two stages. This will avoid any possibility of interaction between input and output impedances directly due to the feedback loop. The difficulty is that it is not easy to employ the large amount of feedback over shorter loops because either the gain is unsufficient, if feedback is taken from the output transformer secondary, or too much power will be absorbed from the plate circuit, if the feedback is taken from the primary.

One step to overcome this difficulty uses an output transformer either with tappings on the primary or a separate winding, in one of the circuits shown at *Fig.* 12. (This is shown single-ended for simplicity; in practice push-pull is used for high-quality work, using the same principles.)

Some single-stage positive current feedback can overcome deficiency of gain and provide impedance reduction. Positive current feedback could be used with the circuit of *Fig.* 13 to produce zero output source impedance without causing instability, provided the output is never short-circuited, but all positive feedback accentuates any distortion present. A compromise, using some positive current feedback combined with negative voltage feedback can achieve zero source impedance without excessive loss of gain, and with reasonable reduction in curvature distortion. The snag is that an extra winding is required on the interstage or driver transformer, and that for push-pull working the output transformer primary halves require to be separated at the center tap.

The extra winding on the interstage transformer is quite small, as it has practically no power to transfer, behaving in conjunction with the rest of the transformer as a current transformer of very high ratio, using the plate resistance of the previous stage to develop the fed back voltage at the grids. Having quite few turns, it can easily be wound on by hand with the older types of interstage job, where there is any room at all to spare.

Manufacturers already make lines of output transformer with provision for feedback, using either tappings or separate windings. It is suggested that drive or interstage types could also be introduced with a similar provision for the above purpose. The exact amount of positive feedback can be adjusted, where the number of turns is more than necessary, for the circuit used, by the arrangement shown in *Fig.* 14, without appreciably increasing losses anywhere.

CONTOUR SELECTOR

(*from page 32*)

system to satisfy his particular taste. Since there are still so many variables involved in high-fidelity system design, not the least of which is the fact that the amplifier may be used with any of a variety of loudspeaker combinations which differ widely in frequency characteristics, it is quite likely that the record equalizer setting may be used primarily as a point of departure for operation of the tone controls.

In the design of the DB20 we attempted to steer a safe course between the Scylla of not enough control for the sophisticate and the Charybdis of alienating his wife (who probably objected to investing in a hi-fi system when what they really needed was a new fur coat for her) by making the whole business of hi-fi too confusing. Our record equalizer was designed with this in mind, and we were on the point of simplifying the unit by providing either (but not both) a volume control or a so-called compensated loudness control when further study changed our thinking.

The Fletcher-Munson equal loudness curves, although they hold only for pure tones and not for the complex sounds