APPENDIX 4.B

SKIN AND PROXIMITY EFFECTS IN HIGH-FREQUENCY TRANSFORMER WINDINGS

4.B.1. INTRODUCTION

The information presented here provides some explanation and justification for the design methods used in Chap. 4. For a more complete background, see References 1, 2, 8, 15, 31, 58, 59, 60, 65, 66, and 67.

To optimize the efficiency of high-frequency switchmode transformers, suitable wire gauges, strip sizes, and winding geometries must be used. Filling up the available window area with a gauge of wire that will fit simply will not do if optimum efficiency is to be obtained. The simple design rules used for line-frequency transformers are inadequate for optimum design of high-frequency transformers.

Figures 3.4B.1, 3.4B.5, and 3.4B.6 show how the effective ac resistance of a winding is related to frequency, wire size, and number of layers. Hence, at high frequencies, when two or three layers of wire are used in a winding, the F_r ratio (the ratio of the effective ac resistance of the winding to its DC resistance) could quite easily be a factor of 10 or more in a poor design. That is, the effective resistance of the winding at the working frequency could be 10 times greater than its DC resistance. This would give excessive power loss and temperature rise.

The intuitive temptation to use as large a wire as possible often leads to the wrong result in a high-frequency application. Using too large a wire results in excessive loss as a result of skin and proximity effects. Hence too large a wire gauge, giving many layers and excessive buildup, is just as inefficient as having too small a gauge. It will be shown that because of skin and proximity effects, an ideal wire size or strip thickness exists, and this must be used if optimum efficiency is to be obtained.

A brief examination of skin and proximity effects would perhaps be helpful at this stage.

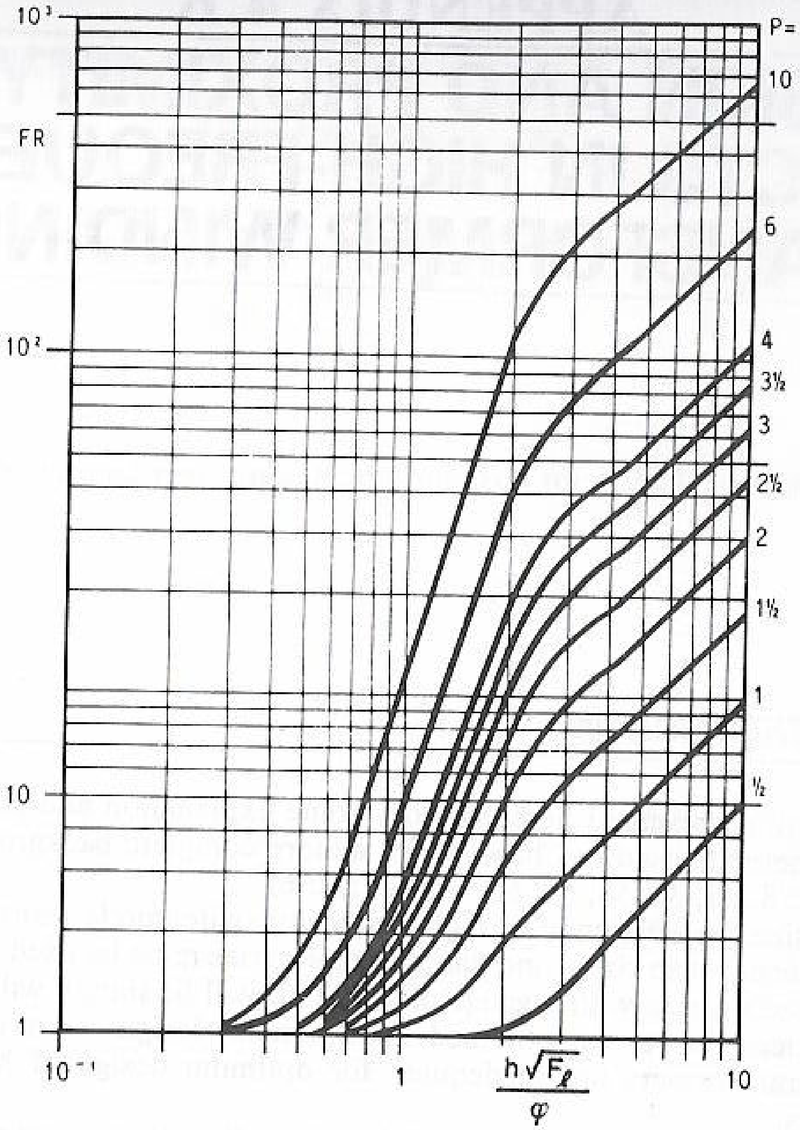

FIG. 3.4B.1 F_r ratio (ratio of ac/DC resistance as a result of skin effect) as a function of the effective conductor thickness, with number of layers P as a parameter (*After Dowell*[31].)

4.B.2. SKIN EFFECT

Figure 3.4B.2 shows how an isolated conductor carrying a current will generate a concentric magnetic field. With alternating currents a magnetomotive force (mmf) exists, generating eddy currents in the conductor. The direction of these eddy currents is such as to add to the current at the surface of the wire and subtract from the current in the center. The effect is to encourage the current to flow near the surface of the conductor (the well-known skin effect). The majority of the current will flow in an equivalent surface skin thickness or penetration depth Δ, defined by the formula

$$\Delta = \frac{K_m}{\sqrt{f}} \tag{4.B.1}$$

where Δ = penetration depth, mm
f = frequency, Hz
K_m = material constant (K_m ranges from 75 for copper at 100°C to = 65.5 for copper at 20°C)

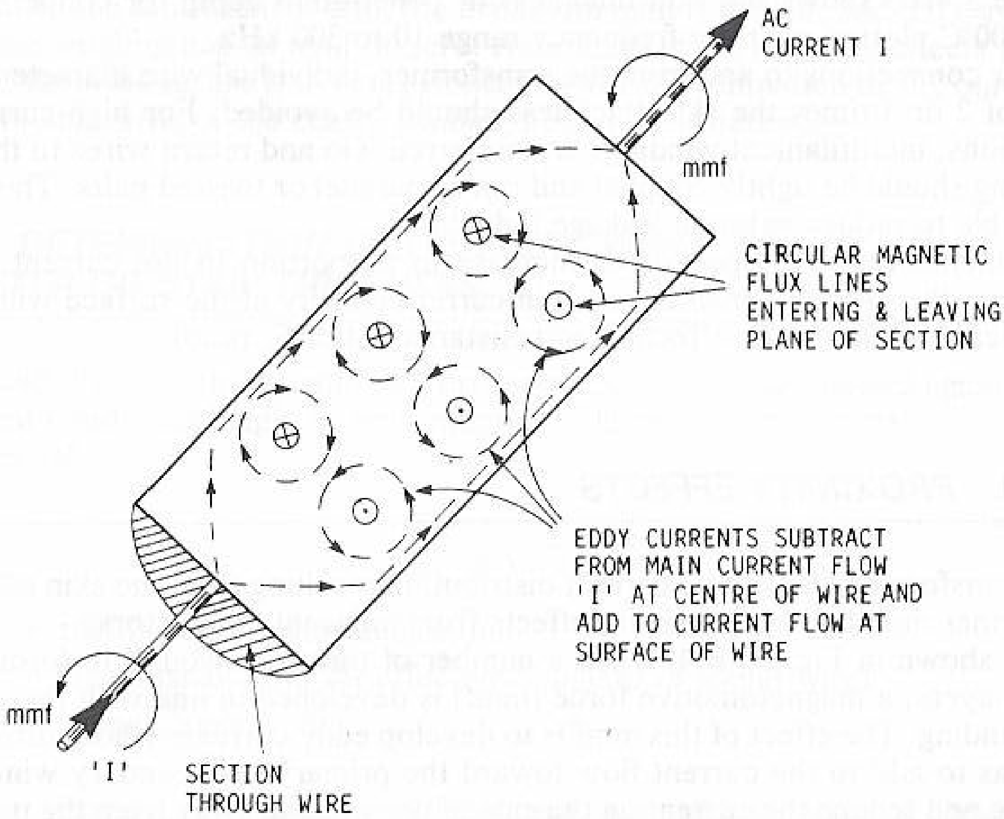

FIG. 3.4B.2 Showing how "skin effect" is caused. Current is constrained to flow in the surface layer of the conductor as a result of concentric magnetic fields in the body of the conductor caused by the current flow.

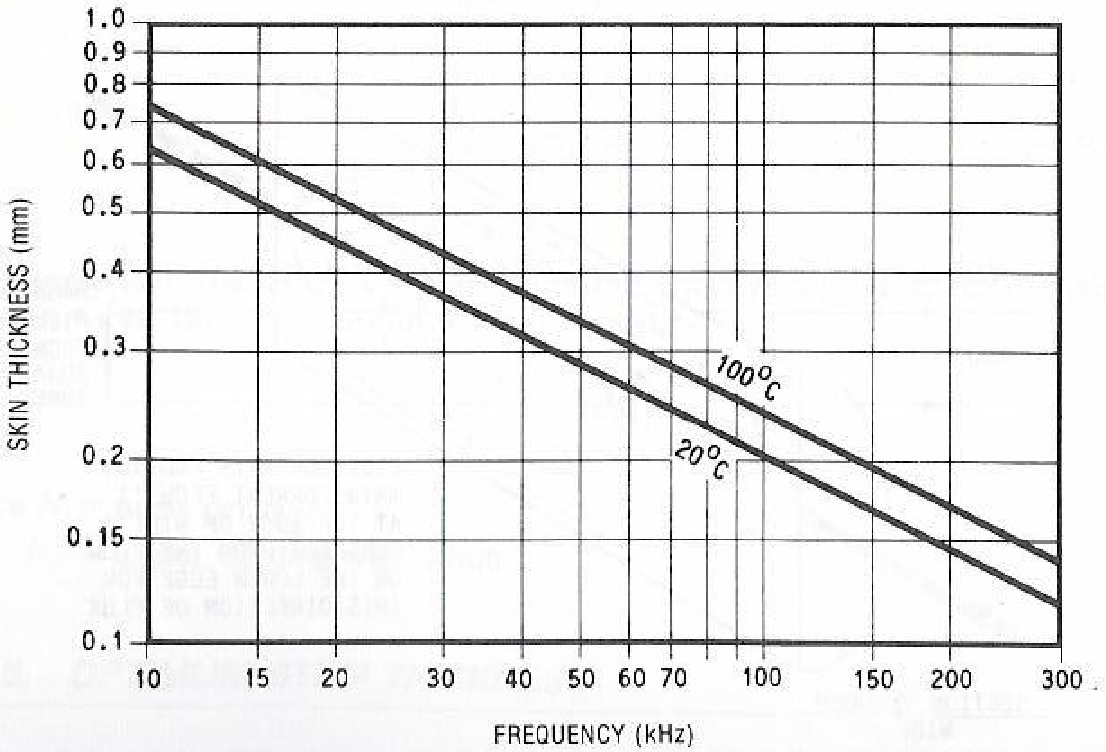

FIG. 3.4B.3 Effective skin thickness as a function of frequency, with temperature as a parameter.

Figure 3.4B.3 shows the skin thickness or penetration depth for copper at 20°C and 100°C plotted over the frequency range 10 to 300 kHz.

For connections to and from the transformer, individual wire diameters in excess of 2 or 3 times the skin thickness should be avoided. For high-current applications, multifilament windings are preferred. Go and return wires to the same winding should be tightly coupled and run as parallel or twisted pairs. This is also desirable to reduce external leakage inductance.

Remember, the copper losses increase in proportion to the current density squared; therefore, a small increase in current density at the surface will have a significant effect on the effective ac resistance ratio (F_r ratio).

4.B.3. PROXIMITY EFFECTS

In a transformer, the simple current distribution resulting from the skin effect will be further modified by proximity effects from adjacent conductors.

As shown in Fig. 3.4B.4, when a number of turns are wound to form one or more layers, a magnetomotive force (mmf) is developed in line with the plane of the winding. The effect of this mmf is to develop eddy currents whose direction is such as to add to the current flow toward the primary-to-secondary winding interface and reduce the current on the side of the winding away from the interface. As a result of these proximity effects, the useful area of a conductor is further reduced.

The proximity effect is most pronounced where the mmf is maximum, that is, at the primary-to-secondary interface. Figure 3.4.8*a* and *b* (p. 3.81) shows the dis-

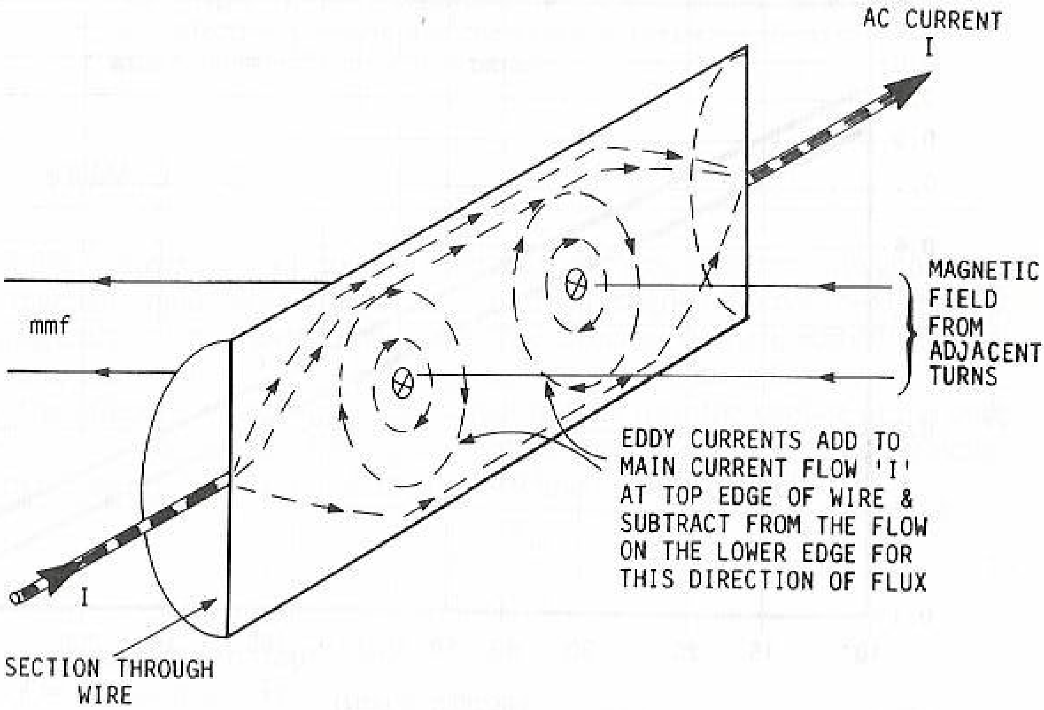

FIG. 3.4B.4 Showing how "proximity effects" are caused. Current is constrained to flow toward the interface of the windings as a result of incident magnetic fields from nearby turns.

tribution of mmf in a simple winding configuration and that in a sandwiched construction. In the sandwiched form, the maximum mmf is halved, and the center of the middle winding has an mmf of zero. As a result, the proximity effects in the center of the winding are also zero. Hence, for the determination of F_r, only half the layers and turns of the center winding need be considered.

4.B.4. DETERMINATION OF OPTIMUM WIRE DIAMETER OR STRIP THICKNESS

Figure 3.4B.1 shows the F_r ratio (ac resistance/DC resistance) plotted against the equivalent conductor height φ, with number of layers as a parameter.

In general,

$$\varphi = \frac{h}{\Delta\sqrt{F_l}} \tag{4.B.2}$$

where φ = effective conductor height, mm

h = thickness of strip (or effective diameter of round wire)

F_l = copper layer factor

Note: To simplify the mathematical treatment, a round conductor of diameter d is replaced by a square one of the same area with an effective thickness h; e.g.,

$$\text{Area of round wire} = \pi r^2 \quad \text{or} \quad \pi(d/2)$$

$$\text{Area of square wire} = h^2$$

Hence

$$\pi\left(\frac{d}{2}\right)^2 = h^2$$

Therefore,

$$h = d\sqrt{\frac{\pi}{4}} \tag{4.B.3}$$

The copper layer factor F_l is a function of the effective wire diameter, spacing between wires, turns, and useful winding width:

$$F_l = \frac{N \cdot h}{b_w}$$

where N = turns per layer

b_w = useful winding width, mm

4.B.5. OPTIMUM STRIP THICKNESS

In the simple case of a full-width copper strip winding, at a fixed frequency, φ goes to h/Δ, since $F_l = 1$; also, the number of layers is equal to the number of turns.

The ideal strip thickness can now be established from Fig. 3.4B.1 as follows:

$$R_{ac} = F_r \cdot R_{dc}$$

but (for a defined bobbin size and number of turns)

$$R_{dc} = \frac{N \cdot \rho \cdot l}{A} \propto \frac{\rho}{b_w \cdot h}$$

where ρ = resistivity of copper, Ω/cm
A = area of wire, cm^2
l = length of wire, cm
N = turns

Hence

$$R_{ac} = F_r \cdot R_{dc}$$

Therefore

$$R_{ac} \propto F_r \cdot \frac{\rho}{b_w \cdot h} \propto \frac{\rho}{b_w \cdot \Delta} \times \frac{F_r}{\varphi} \qquad (\text{since } \varphi = h/\Delta)$$

and

$$\frac{R_{ac} \cdot b_w \cdot \Delta}{\rho} \propto \frac{F_r}{\varphi} \tag{4.B.4}$$

By plotting F_r/φ against φ with turns (or layers with a strip winding) as a parameter (Fig. 3.4B.5), the minimum ac resistance point for each number of turns can be seen. The minimums fall close to a line (dashed in the figure) where F_r = 4/3. For a given number of turns (layers), the optimum strip thickness h is obtained from the lower scale as a multiple of the skin thickness at the operating frequency. For example, with two turns, the minimum ac resistance is found where H/Δ = 1 and the optimum strip thickness is the same as the skin thickness.

It has been shown that for a simple strip winding, the optimum strip thickness is a function of the frequency and number of turns (layers), and will be found near an F_r ratio of 1.33.

4.B.6. OPTIMUM WIRE DIAMETER

With round wire windings, the determination of optimum wire diameter is more complex than that for the strip winding, shown above; however, by a similar process, it has been shown[58,59,60,66] that the optimum wire diameter will be found near F_r = 1.5.

Figures 3.4B.6 and 3.4B.7 show the optimum copper strip thickness or wire diameter to use to obtain the ideal F_r ratio. They are plotted against the number of "effective layers," with frequency as a parameter.

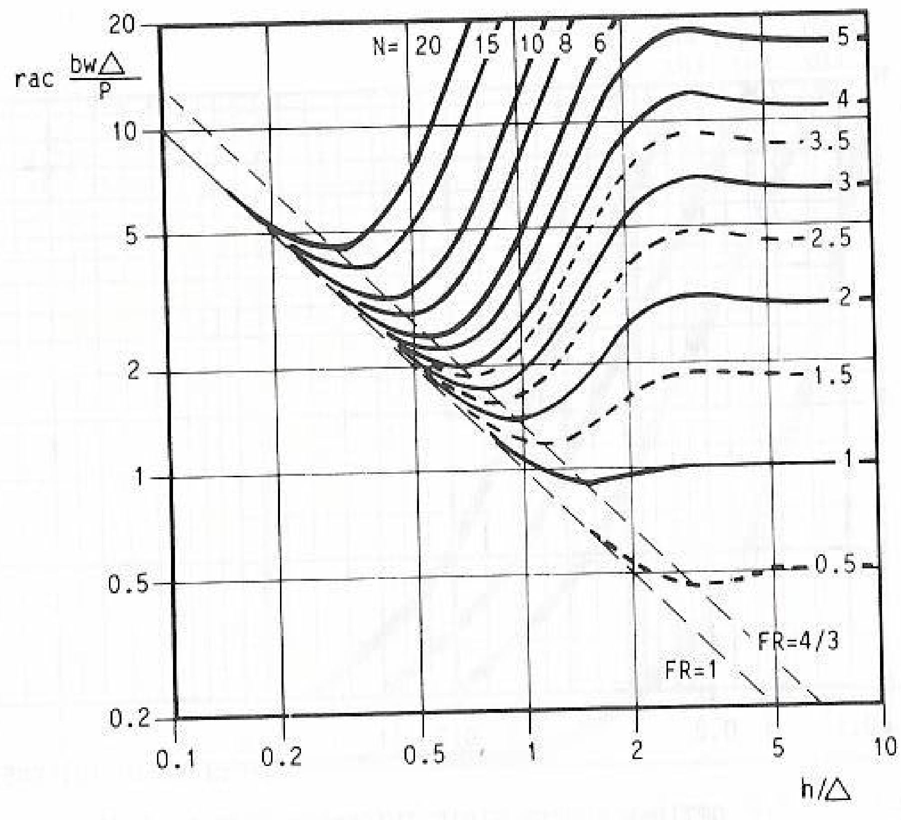

FIG. 3.4B.5 Plot of R_{ac-h}/Δ with number of layers as a parameter, showing the development of the conditions for optimum F_r ratio and optimum conductor thickness. (*J. Jongsma 1882 Mullard Ltd. Ref. 58.*)

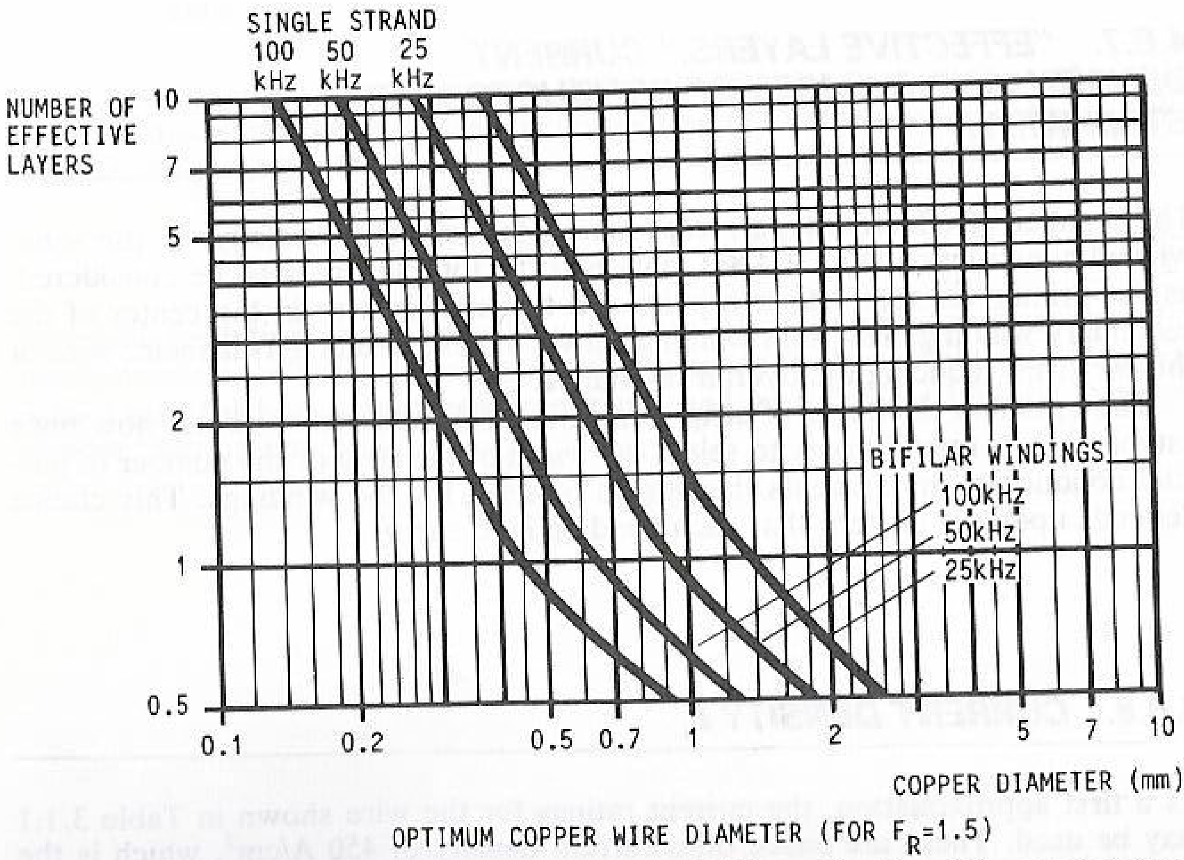

FIG. 3.4B.6 Optimum wire diameter for an F_r ratio of 1.5, as a function of the number of effective layers in the winding, with frequency as a parameter (*Mullard Ltd.*)

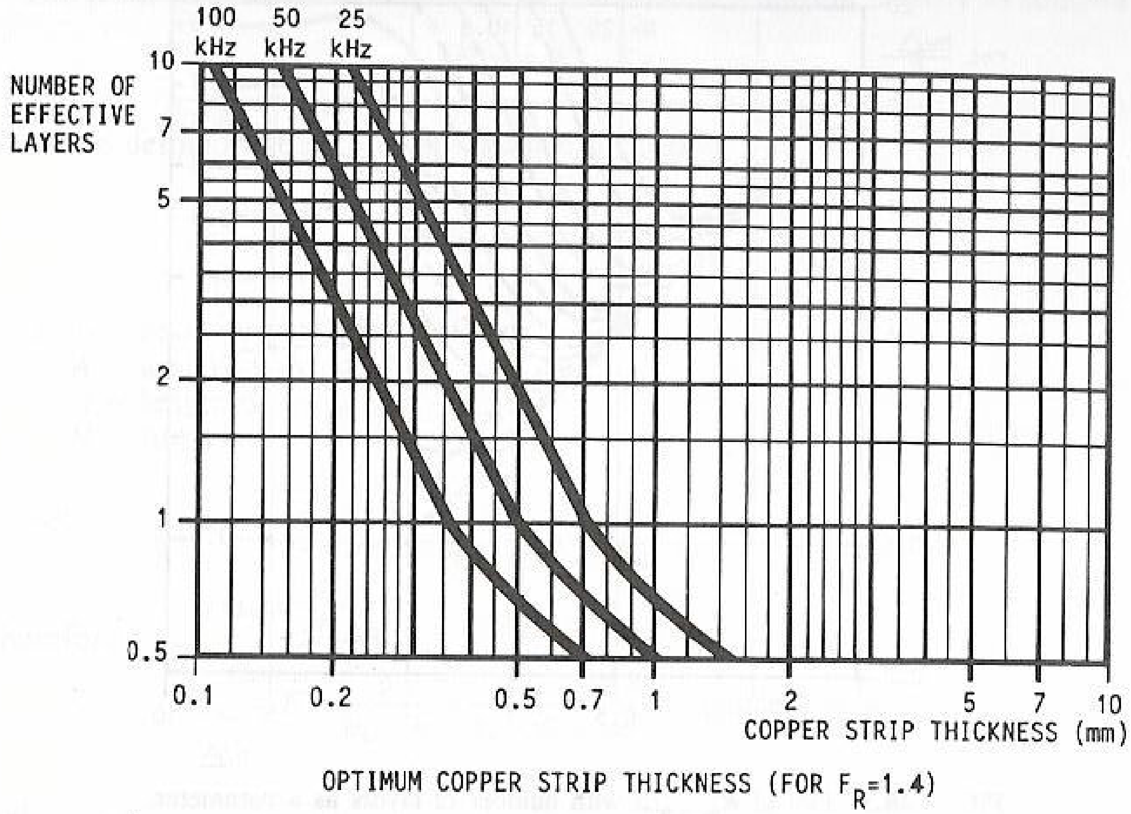

FIG. 3.4B.7 Optimum strip thickness for an F_r ratio of 1.4, as a function of the number of effective layers, with frequency as a parameter. (*Mullard Ltd.*)

4.B.7. "EFFECTIVE LAYERS," CURRENT DENSITY, AND NUMBER OF CONDUCTORS OR STRIP WIDTH

The number of effective layers depends on the winding topology. In the sandwiched construction (Fig. 3.4.8*c*), only half the total layers need be considered, as the primary is split into two parts and the mmf is zero in the center of the secondary winding. This split winding allows the use of a larger-diameter wire or thicker strip, reducing the overall resistance.

The optimum thickness of strip or maximum diameter of wire has now been established. It remains only to select the width of the strip or the number of parallel conductors or filaments that are to be used for the windings. This choice depends upon the current that each winding is to carry.

4.B.8. CURRENT DENSITY I_a

As a first approximation, the current ratings for the wire shown in Table 3.1.1 may be used. These are based on a current density of 450 A/cm^2, which is the optimum density for a core with an area product of 1 cm^4 and a 30°C temperature rise.

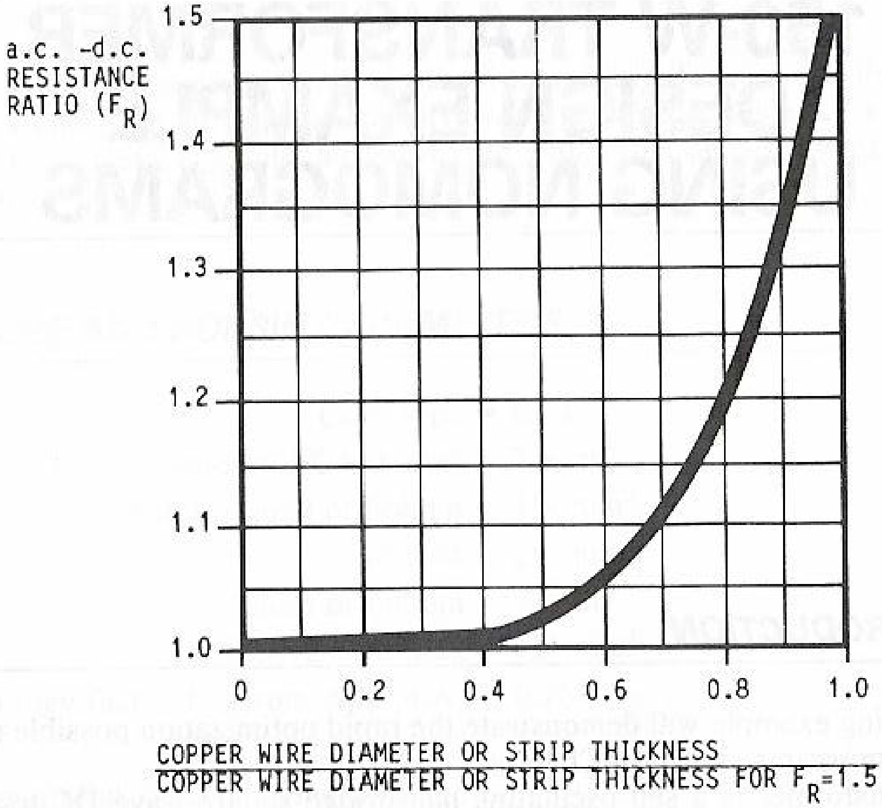

FIG. 3.4B.8 F_r ratio for wires below optimum thickness. (*Mullard Ltd.*)

More correctly, with larger cores the current density I_a should be reduced, as the heat-dissipating surface area increases less rapidly than the heat-generating volume. In general:

$$I_a = 450 \cdot AP^{-0.125} \qquad \text{A/cm}^2 \tag{4.B.5}$$

Practical limitations may not allow the use of the optimum wire size. For nonoptimum conditions, the effective ac resistance may be established from Fig. 3.4B.1 for a thickness greater than optimum, and from Fig. 3.4B.8 for a thickness smaller than optimum.